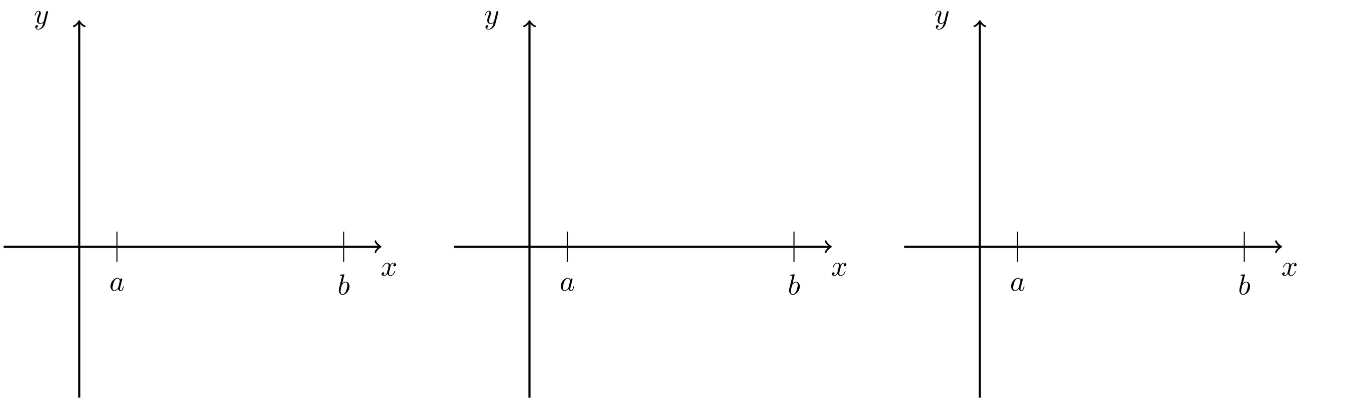
4-2 The Mean Value Theorem

Motivating Examples: Draw several examples of graphs of functions such that (i) the domain is $[a, b]$ and (ii) $f(a) = f(b)$. Note you are not *required* to make sketches that are continuous or differentiable, though you may choose to do so.



Draw some conclusions:

Rolle's Theorem: If ____________________,

then there is a number c in the interval (a, b) such that $f'(c) = 0$.

Proof:

PRACTICE PROBLEMS:

1. Consider $f(x) = x^4 - \frac{8}{3}x^3 + 1$ on the interval $[0, 8/3]$.

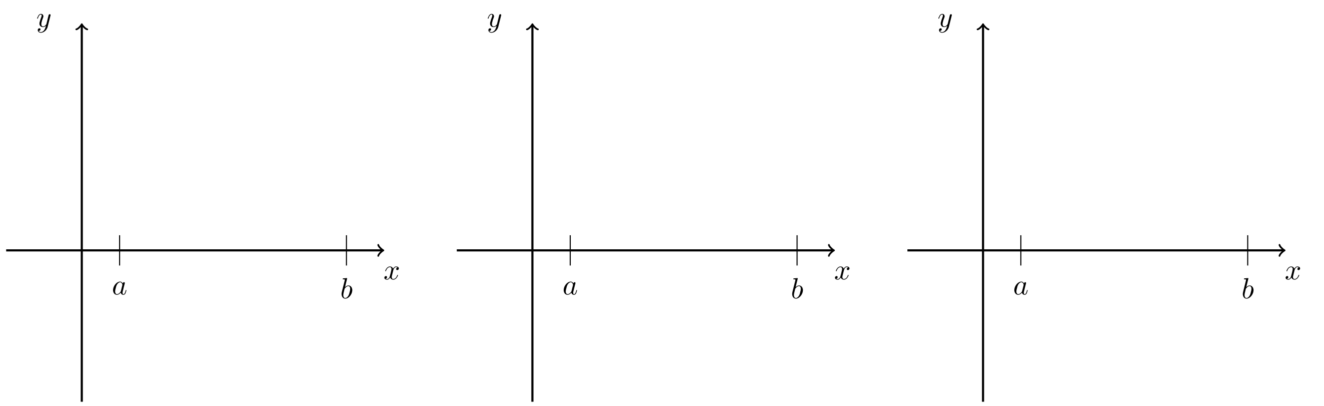
   (a) Verify that the function $f(x)$ satisfies the hypothesis of Rolle's Theorem on the given interval.

   (b) Find all numbers c that satisfy the conclusion of Rolle's Theorem.

   (c) Sketch the graph on your calculator to show that your answer above are correct.

2. *Use Rolle's Theorem* to show that the equation $x^3 - 15x + d = 0$ can have at most one solution in the interval $[-2, 2]$.
   HINT: Show that there is no way there could be two solutions!

MOTIVATING EXAMPLES: Draw several examples of graphs of functions such that (i) the domain is $[a, b]$, (ii) $f(x)$ is continuous on $[a, b]$, **and** (iii) $f(x)$ is differentiable on $[a, b]$. We are *not* assuming that $f(a) = f(b)$.



QUESTION: In each picture above, draw (or in some other way identify) the quantity:

$$\frac{f(b) - f(a)}{b - a}.$$

What would this quantity be if Rolle's Theorem applied?

THE MEAN VALUE THEOREM: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is a number c in the interval (a, b) such that

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QUESTION: Assume that $f(x)$ is continuous and differentiable on the interval $[a, b]$ and assume there exists some x-value d in (a, b) such that $f(d) > f(a)$, can you draw any conclusion about $f'(x)$? What if $f(d) < f(a)$?

THEOREM 5: If $f'(x) = 0$ for all x in the interval (a, b), then

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QUESTION: If $f(x)$ gives the *position* of an object as a function of time, what "common sense" idea is the MVT telling us? Theorem 5?

1. Sketch the graph of a function $f(x)$ on an interval $[0, 5]$ such that there are exactly three numbers c in $(0, 5)$ satisfying the Mean Value Theorem.

2. Suppose that $f(0) = -3$ and that $f'(x)$ exists and is less than or equal to 5 for all values of x. How large can $f(2)$ possibly be?

3. Consider $f(x) = x^{-2}$ on the interval $[1, 2]$.

   (a) Verify that the function $f(x)$ satisfies the hypothesis of the Mean Value Theorem on the given interval.

   (b) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

   (c) Sketch the graph to show that your answer above are correct.

ONE LAST BIG IDEA:

1. Give the formulas for two *different* functions $f(x)$ and $g(x)$ such that $f'(x) = g'(x)$ and sketch these two functions on the same set of axes.

2. **Corollary 7:** If $f'(x) = g'(x)$ for all x in the interval (a, b), then

   or, said another way,

   .

3. Why is Corollary 7 true?