LECTURE NOTES: 4-5 CURVE SKETCHING (PART 2)

WARM UP PROBLEM Find your copy of the Graphing Guidelines!

PRACTICE PROBLEMS

1. Sketch the curve $y = x - 2\sin x$ on $[-2\pi, 2\pi]$.

   (a) Find the domain. $\mathbb{R}$

   (b) Find the x and y-intercepts.

   when $x=0$, $y=0$.
   when $y=0$,... solve $2\sin x = x$? hard. let it go.

   Hey! once we are done we see there sure are other solutions. We'll find them in §4.8

   (c) Find the symmetries/ periodicity of the curve.

   x, $\sin x$ both odd.

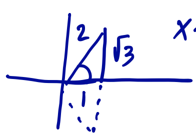
   So I expect the function to be odd.

   (d) Determine the asymptotes. none. $\lim_{x\to\infty} x - 2\sin x = \infty$, $\lim_{x\to-\infty} x - \sin x = -\infty$.

   (e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$y' = 1 - 2\cos x = 0$

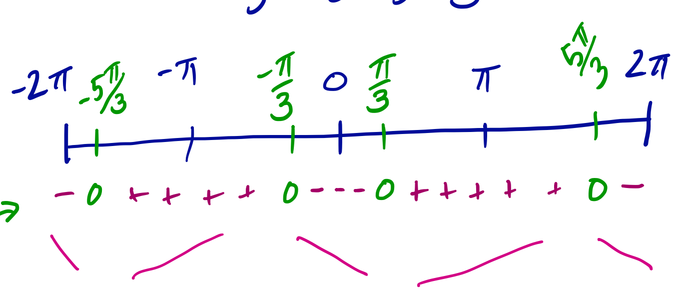
$\cos x = \frac{1}{2}$



$x = \frac{\pi}{3}, -\frac{\pi}{3}$

critical points in $[-2\pi, 2\pi]$ are:

$x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$



Sign of y'

ANS:

y is increasing on $\left(-\frac{5\pi}{3}, -\frac{\pi}{3}\right) \cup \left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$

and decreasing on $\left(-2\pi, -\frac{5\pi}{3}\right) \cup \left(-\frac{\pi}{3}, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right)$.

local minimums at $x = -\frac{5\pi}{3}$, min value $-\frac{5\pi}{3} - \sqrt{3}$

at $x = \frac{\pi}{3}$, min value $\frac{\pi}{3} - \sqrt{3}$

at $x = 2\pi$, min value 2π

local maximums at $x = -2\pi$, max value -2π

at $x = -\frac{\pi}{3}$, max value $-\frac{\pi}{3} + \sqrt{3}$

at $x = \frac{5\pi}{3}$, max value $\frac{5\pi}{3} + \sqrt{3}$

(g) Find the intervals of concavity/inflection points.

$y' = 1 - 2\cos x$

So $y'' = 2\sin x$.

So $y'' = 0$ in $[-2\pi, 2\pi]$

when $x = -2\pi, -\pi, 0, \pi, 2\pi$

-2π		$-\pi$		0		π		2π	
0	+	0	−	0	+	0	−	0	← sign of y''

<u>answer</u>:

y is concave up on $(-2\pi, \pi) \cup (0, \pi)$ and

concave down on $(-\pi, 0) \cup (\pi, 2\pi)$.

inflection points:

x	$-\pi$	0	π
y	$-\pi$	0	π

(h) Sketch the curve.

points to plot:

$(-2\pi, -2\pi)$ ✓

$\left(-\frac{5\pi}{3}, \approx -7\right)$ ✓

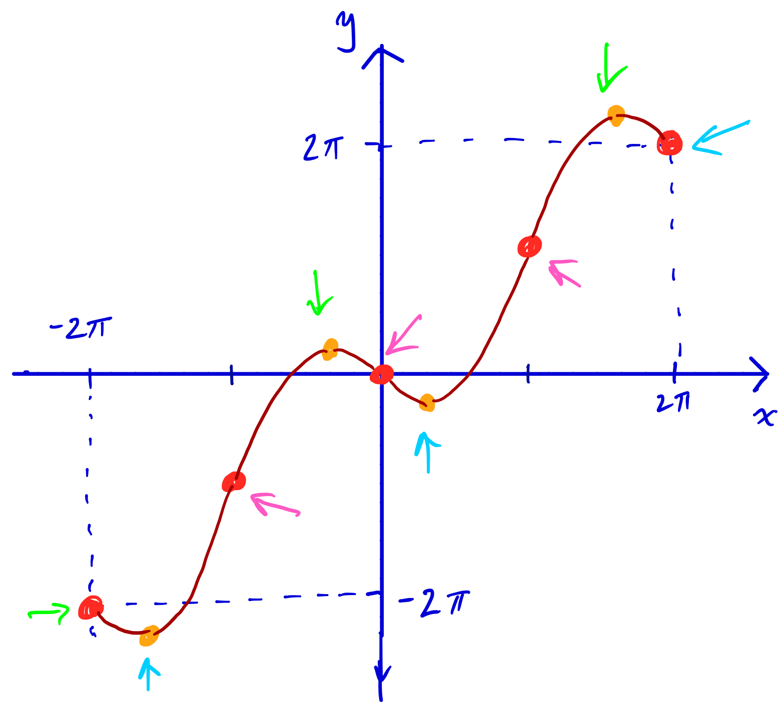
$\left(-\frac{\pi}{3}, \approx 0.69\right)$ ✓

$(0, 0)$ ✓

$\left(\frac{\pi}{3}, \approx -0.69\right)$ ✓

$\left(\frac{5\pi}{3}, \approx 7\right)$ ✓

$(2\pi, 2\pi)$ ✓



• local max pts • inflection pts

• local min pts

4. Sketch the curve $y = \dfrac{x}{\sqrt{9+x^2}}$

(a) Find the domain. $\mathbb{R}$

(b) Find the x and y-intercepts.

$(0,0)$

(c) Find the symmetries/ periodicity of the curve.

odd

(d) Determine the asymptotes. no vertical asymptotes.

$\lim_{x\to\infty} \dfrac{x}{\sqrt{9+x^2}} = 1$. So $y=1$ is a horizontal asym.

tricky! $\lim_{x\to -\infty} \dfrac{x}{\sqrt{9+x^2}} = -1$. So $y=-1$ is a horizontal asymp.

(e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$y' = 9(x^2+9)^{-3/2}$

So $y' > 0$ always.

answer: y is always increasing.
y has no local max's or mins.

(g) Find the intervals of concavity/inflection points.

$y'' = \dfrac{-27x}{(x^2+9)^{5/2}}$

answer y is concave up on $(-\infty, 0)$ and concave down on $(0,\infty)$.
The point $(0,0)$ is an inflection point.

$y''=0$ when $x=0$.
$y''>0$ when $x<0$; $y''<0$ when $x>0$

(h) Sketch the curve.