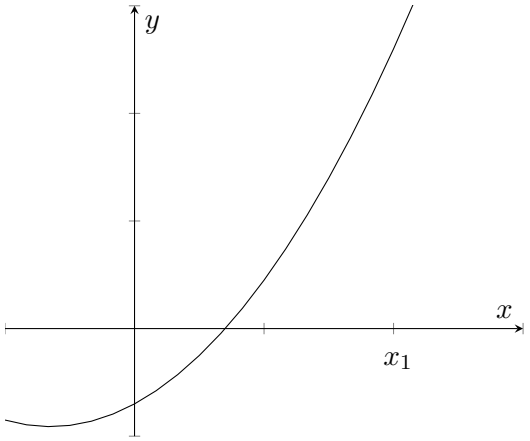
4-8

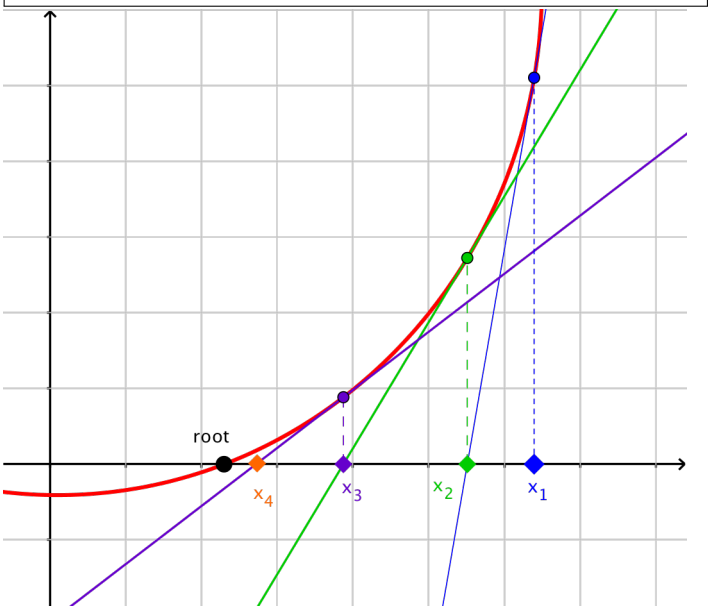
MOTIVATING QUESTION: Recall that we wanted to find the x-intercepts of $f(x) = x - 2\sin x$. From the graph we knew there exists a positive (and negative) solution. How to find it?

1. Write the equation of the line tangent to the curve $y = f(x)$ at the x-value x_1. Sketch the tangent line in the "cartoon" of $f(x)$ below.



2. In your picture above, label the x-value where the tangent line intersects the x-axis as x_2.
3. Solve for x_2 using your equation from part (1) above.

GEOMETRIC EXPLANATION OF NEWTON'S METHOD:



MODEL PROBLEM: Let $f(x) = x^3 - 5x$.

1. Factor $f(x)$, find its roots algebraically, and sketch its graph.

2. Assume you couldn't factor the function and you wanted to find its positive root. What would be a reasonable first guess and why?

3. Using a first guess of $x_1 = 3$, calculate 2 iterations of Newton's method

4. How close is your estimate of the root, x_3, to the actual root?

QUESTIONS FOR YOU:

1. In general, when applying Newton's Method can there exist bad choices for x_1? Explain.

2. Use Newton's Method to approximate any zeros of $g(x) = x - 2\sin x$ accurate to at least 9 decimal places. [NOTE: The FIRST thing you must do is make a guess at where that root might be. Make it a GOOD guess.]

3. Estimate $\sqrt[6]{7}$ correct to 5 decimal places. [Note: *you* must construct an appropriate $f(x)$ here.]

4. Explain how Newton's Method could be used to find points of intersections between curves, say where $f(x) = -x/3$ and $g(x) = \ln x$ intersect.