4-3 Routine Problems

1. Given the function $f(x) = \ln(x^2 + 4)$ find the following. *For parts a-d, put your answer in a box.*

   (a) Determine the domain of $f(x)$. Since $x^2+4>0$, domain $\boxed{(-\infty,\infty)}$

   (b) Find the intervals of increase or decrease.

$$f'(x) = \frac{2x}{x^2+4}$$

− 0 + ← sign f′

−1 0 1 ← sample points

c.p. $x=0$

answer:
$\boxed{\text{f is increasing on } (0,\infty) \text{ and decreasing on } (-\infty,0)}$

   (c) Find the local maximum and minimum values.

$\boxed{\text{f has a local min of } f(0)=\ln 4 \text{ at } x=0 \text{. f has no local max.}}$

   (d) Find the intervals of concavity and inflection points.

$$f''(x) = \frac{2(2-x)(2+x)}{(x^2+4)^2}$$

f is concave up on $(-2,2)$ and concave down $(-\infty,-2)\cup(2,\infty)$

$f''=0$ when $x=2$ or $x=-2$

- - - - 0 + + + 0 - - - ← sign f″

−3 −2 0 2 3 ← sample points

Inflection points

$(-2, f(-2)) = (-2, \ln 8)$

$(2, f(2)) = (2, \ln 8)$

   (e) Use the information to sketch the graph.

Putting thing together

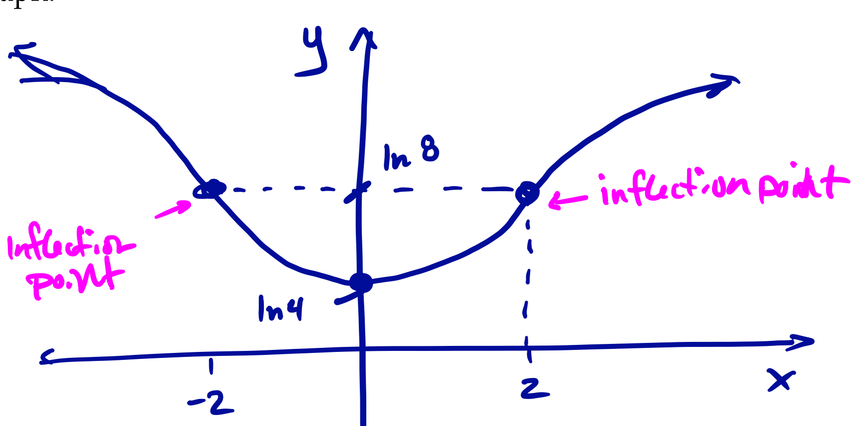
− − − 0 + + + ← sign f′

−2 0 2

- - - 0 + + + + 0 - - - ← sign f″

↓, ccdown | ↓ccup | ↑ccup | ↑ccdown

← shape



2. Sketch a possible graph of a function f that satisfies the following conditions:

   (a) f is continuous and differentiable on $(-\infty, \infty)$.

   (b) $f(0) = 2$, $f(2) = 3$, $f(4) = 2$

   (c) $f'(2) = 0$

   (d) $f'(x) > 0$ for $x < 2$ and $f'(x) < 0$ for $2 < x$

   (e) $f''(x) > 0$ for $4 < x$ and $f''(x) < 0$ for $x < 4$.