1. (REVIEW)

   (a) Find the most general antiderivative of $f(x) = \sqrt{2} - e^x + 4\cos x$.

   $F(x) = \sqrt{2}x - e^x + 4\sin x + C$

   use this here

   (b) Find $g(x)$ if $g'(x) = \sqrt{2} - e^x + 4\cos x$ and $\boxed{g(\pi) = 1}$.

   $g(x) = \sqrt{2}x - e^x + 4\sin x + C$ $\quad$ $\sin\pi = 0$

   $1 = g(\pi) = \sqrt{2}\pi - e^\pi + 4\sin\pi + C$

   $C = 1 - \sqrt{2}\pi + e^\pi$; Answer: $g(x) = \sqrt{2}x - e^x + 4\sin x + 1 + e^\pi - \sqrt{2}\pi$

   (c) If $g'(x)$ represented velocity, what is $g(x)$? What would $g(0)$ mean? $g(2)$?

   $g(x)$ would give position.

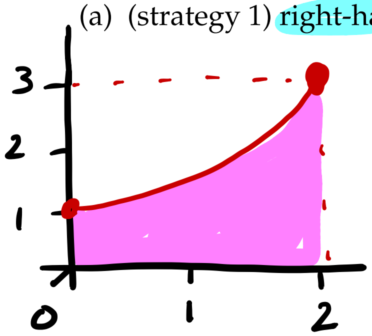
   $g(0)$ position when time starts.

   $g(2)$ position of object when time is 2.

Section 5.1

2. Goal of this next part is to estimate the area under the curve $y = \frac{1}{2}x^2 + 1$ and above the x-axis on the interval $[0, 2]$.

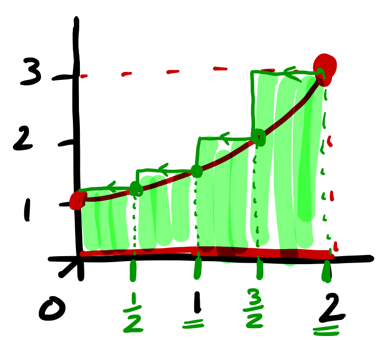
   (a) (strategy 1) right-hand endpoints, n=4 rectangles



$y(0) = 1$

$y(2) = \frac{4}{2} + 1 = 3$

Idea: area pink $\approx$ area green $<$

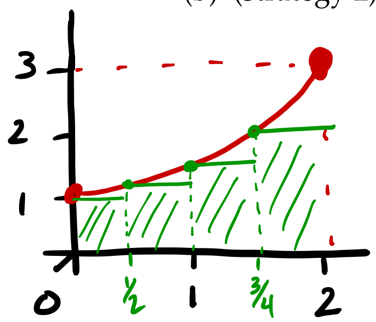


area under $y = \frac{1}{2}x^2 + 1$ on $[0,2]$ $\approx R_1 + R_2 + R_3 + R_4 = \frac{1}{2} \cdot f(\frac{1}{2}) + \frac{1}{2} f(1) + \frac{1}{2} f(\frac{3}{2}) + \frac{1}{2} f(2)$

$= \frac{1}{2}\left[\frac{1}{2}(\frac{1}{2})^2 + 1 + \frac{1}{2}(1)^2 + 1 + \frac{1}{2} \cdot (\frac{3}{2})^2 + 1 + \frac{1}{2}(2)^2 + 1\right]$

$= 3.875$

(b) (strategy 2) left-hand endpoints, n=4 rectangles   2.875



area green < area pink

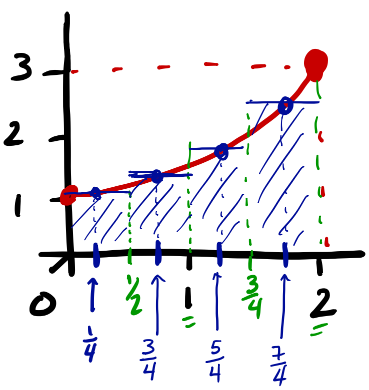
area green $= R_1+R_2+R_3+R_4$

$$= \frac{1}{2}\left[f(0)+f\left(\tfrac{1}{2}\right)+f(1)+f\left(\tfrac{3}{4}\right)\right]$$

$$= \frac{1}{2}\left[\left(\tfrac{1}{2}0^2+1\right)+\left(\tfrac{1}{2}\left(\tfrac{1}{2}\right)^2+1\right)+\left(\tfrac{1}{2}1^2+1\right)+\tfrac{1}{2}\left(\tfrac{3}{4}\right)^2+1\right]$$

$$= 2.875$$

(c) (strategy 3) midpoints, n=4 rectangles   3.3125



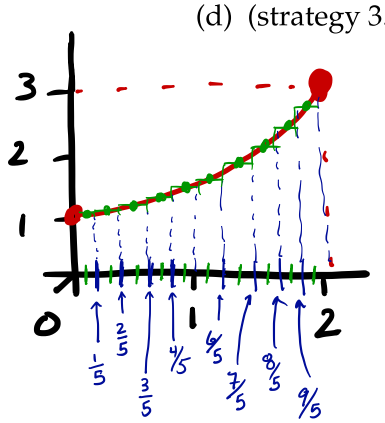
blue area $= R_1+R_2+R_3+R_4$

$$= \frac{1}{2}\left[f\left(\tfrac{1}{4}\right)+f\left(\tfrac{3}{4}\right)+f\left(\tfrac{5}{4}\right)+f\left(\tfrac{7}{4}\right)\right]$$

$$= \frac{1}{2}\left[\left(\tfrac{1}{2}\left(\tfrac{1}{4}\right)^2+1\right)+\left(\tfrac{1}{2}\left(\tfrac{3}{4}\right)^2+1\right)+\left(\tfrac{1}{2}\left(\tfrac{5}{4}\right)^2+1\right)+\left(\tfrac{1}{2}\left(\tfrac{7}{4}\right)^2+1\right)\right]$$

$$= 3.3125$$

(d) (strategy 3.1) midpoints, n=10 rectangles   3.33



width of rectangles $\frac{2}{10} = \frac{1}{5}$

midpoints: (in green) $\frac{1}{10}, \frac{3}{10}, \frac{5}{10}, \frac{7}{10}, \frac{9}{10}, \frac{11}{10}, \frac{13}{10}, \frac{15}{10}, \frac{17}{10}, \frac{19}{10}$

$$\text{area} = \sum_{k=1}^{10} R_i$$

$$= \frac{1}{5}\left[f\left(\tfrac{1}{10}\right)+f\left(\tfrac{3}{10}\right)+f\left(\tfrac{5}{10}\right)+f\left(\tfrac{7}{10}\right)+f\left(\tfrac{9}{10}\right)+f\left(\tfrac{11}{10}\right)+f\left(\tfrac{13}{10}\right)+f\left(\tfrac{15}{10}\right)+f\left(\tfrac{17}{10}\right)+f\left(\tfrac{19}{10}\right)\right]$$

$$= 3.33$$

3. Suppose the odometer on our car is broken and we want to estimate the distance driven over a 1.5 hour time period. We take speedometer readings every 15 minutes and then record them in the table below. Estimate the distance traveled by the car. What method are you using?

I_1 I_2 I_3 I_4 I_5 I_6 ← Six 15 minute intervals (15 min = $\frac{1}{4}$ hr)

Time (minutes)	0	15	30	45	60	75	90
Velocity (mi/h)	17	21	24	29	32	31	28

estimate 1 (using right-hand enpts) $= \frac{1}{4}\left[21+24+29+32+31+28\right] = 41.25$ miles.

estimate 2 (left-hand endpoints) $= \frac{1}{4}\left[17+21+24+29+32+31\right] = 38.5$ miles

4. Oil leaked out of a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed and values of the rate at 2 hour time intervals are shown in the table. Estimate how much oil leaked out. What method are you using? Is it an overestimate or an underestimate.

I_1 I_2 I_3 I_4 I_5 ← five 2-hour intervals

t (h)	0	2	4	6	8	10
r(t) (L/h)	8.7	7.6	6.8	6.2	5.7	5.3

(right-hand) estimate 1 $= 2\left[7.6+6.8+6.2+5.7+5.3\right] = 63.2$ liters

(left-hand) estimate 2 $= 2\left[8.7+7.6+6.8+6.2+5.7\right] = 70$ liters.