1. Find df/dx.

   (a) $f(x) = \left(x^3 + \cos x\right)^2$

   $$\frac{df}{dx} = \underbrace{2(x^3+\cos x)}_{\frac{df}{du}}\underbrace{(3x^2-\sin x)}_{\frac{du}{dx}}$$

   (b) $f(x) = \int_1^{x^3+\cos x} te^t\, dt$

   $$\frac{df}{dx} = \underbrace{\left((x^3+\cos x)\, e^{x^3+\cos x}\right)}_{\frac{df}{du}}\underbrace{(3x^2-\sin x)}_{du/dx}$$

2. Find:

   (a) $\int \left(\frac{e^z}{4} + \frac{1}{1+z^2} - \frac{1}{2}\right) dz$ $= \frac{1}{4}e^z + \arctan z - \frac{1}{2}z + C$

   (b) $\int_1^2 \frac{1}{z^2} - \frac{2}{z^3}\, dz$ $= \int_1^2 \left(z^{-2} - 2z^{-3}\right) dz = -z^{-1} + z^{-2}\Big]_1^2$

   $$= \left(-(2)^{-1} + (2)^{-2}\right) - \left(-(1)^{-1} + 1^{-2}\right)$$

   $$= -\frac{1}{2} + \frac{1}{4} - 0 = -\frac{1}{4}$$

3. The velocity $v(t)$ of an object is given by $v(t) = (t-2)^2 - 1 = t^2 - 4t + 3$ where v is measured in meters per second and t is measured in seconds.

(a) Find and interpret $v(0)$.

$v(0) = 3$

Object going 3 m/s when time starts.

(b) Find and interpret $\int_0^1 \underbrace{v(t)}_{=v}\,\underbrace{dt}_{\Delta t}$ $= \int_0^1 (t^2-4t+3)\,dt = \frac{1}{3}t^3 - 2t^2 + 3t \Big]_0^1$

$\Delta s = v\,\Delta t$

$= \left(\frac{1}{3} - 2 + 3\right) - (0) = \frac{4}{3}$ m.

The object went 4/3 meter in the first second.

(c) Find and interpret $\int v(t)\,dt$ $= \int (t^2-4t+3)\,dt = \frac{1}{3}t^3 - 2t^2 + 3t + C$

So $s(t) = \frac{1}{3}t^3 - 2t^2 + 3t + C$, which gives the position of the object. We don't know it precisely b/c we don't know C.

(d) Now assume you know that when $t = 0$ the object has position $s = 5$ meters. How does this affect your answers to parts (a)-(c) above?

ⓐ This fact doesn't change anything.

ⓑ Now we know that the object started at $S=5$, so when $t=1$, the object is at position $5 + \frac{4}{3} = \frac{19}{3}$ m.

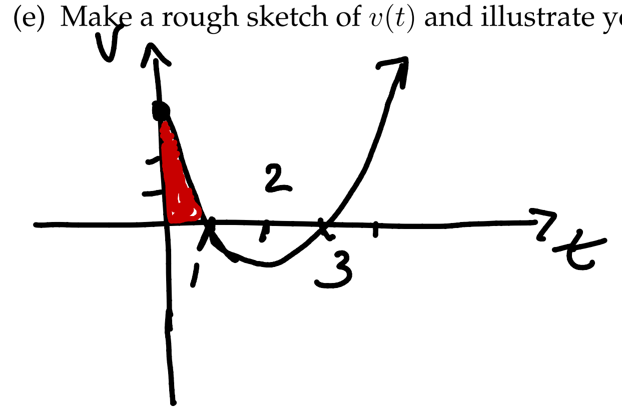
ⓒ We know the precise position function:

$s(t) = \frac{1}{3}t^3 - 2t^2 + 3t + 5$.

$$v(t) = (t-2)^2 - 1$$

$t=0,\ v=3$
$t=1,\ v=0$
$t=2,\ v=-1$
$t=3,\ v=0$

(e) Make a rough sketch of $v(t)$ and illustrate your answer from part (b) above.



$$\int_0^1 v(t)\,dt = \text{area}$$

(f) Find and interpret $\int_0^3 v(t)\,dt$ and $s(3)$.

$S(3) = 5$; $\int_0^3 v(t)\,dt = 0$.

After 3 seconds, the object is back where it started, at position 5m.

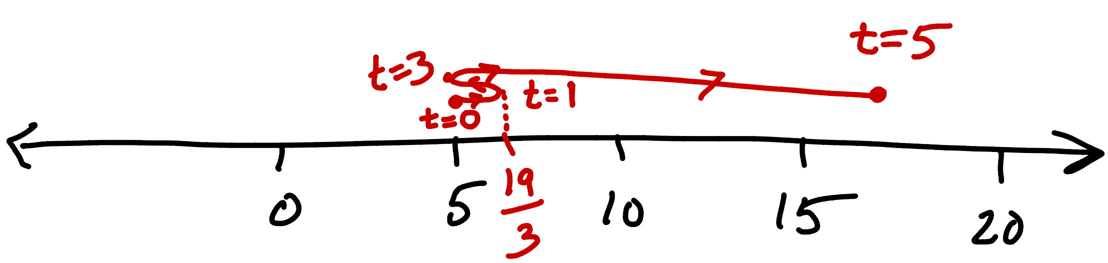
(g) Using $\int_0^5 v(t)\,dt = 20/3$, explain where the object is at $t = 5$.

The object is in position $5 + \frac{20}{3} = \frac{45}{3}$ m.

$$\begin{array}{r} 17.\overline{6} \\ 3\,\overline{)\,53} \\ \underline{3} \\ 23 \\ \underline{21} \end{array}$$

(h) How far did the object travel from $t = 0$ to $t = 5$?

$$\frac{45}{3} + \frac{4}{3} + \frac{4}{3} = \frac{53}{3}\text{ m} \approx 17.6\text{m}$$



(e) Make a rough sketch of $v(t)$ and illustrate your answer from part (b) above.

(f) Find and interpret $\int_0^3 v(t)\,dt$ and $s(3)$.

(g) Using $\int_0^5 v(t)\,dt = 20/3$, explain where the object is at $t = 5$.

(h) How far did the object travel from $t = 0$ to $t = 5$?