

$$\begin{aligned}
 1. \text{ Compute } \int x^2(3-x) dx &= \int (3x^2 - x^3) dx \\
 &= x^3 - \frac{1}{4}x^4 + C
 \end{aligned}$$

$$\begin{aligned}
 2. \text{ Compute } \int (9\sqrt{x} - 3 \sec(x) \tan(x)) dx &= \int (9x^{\frac{1}{2}} - 3 \sec x \tan x) dx \\
 &= 9 \cdot \frac{2}{3} x^{\frac{3}{2}} - \sec x + C \\
 &= 6x^{\frac{3}{2}} - \sec x + C
 \end{aligned}$$

3. Find an antiderivative of $f(x) = \frac{1}{x^2}$ that does not have the form $-1/x + C$.

$$f(x) = x^{-2}, \quad F(x) = -x^{-1} \longrightarrow \text{oh, yeah, right...}$$

$$F(x) = \begin{cases} -x^{-1} + 1 & \text{for } x > 0 \\ -x^{-1} + 10 & \text{for } x < 0 \end{cases}$$

4. Snow is falling on my garden at a rate of

$$A(t) = 10e^{-2t}$$

kilograms per hour for $0 \leq t \leq 2$, where t is measured in hours.

- a. If $m(t)$ is the total mass of snow on my garden, how are $m(t)$ and $A(t)$ related to each other?

$$m'(t) = A(t)$$

- b. What does $m(2) - m(0)$ represent?

How much snow fell in the first two hours in kg.

- c. Find an antiderivative of $A(t)$.

$$F(t) = -5e^{-2t} \quad (+C)$$

any constant including 0 will be ok.

- d. Compute the total amount of snow accumulation from $t = 0$ to $t = 1$.

$$F(1) - F(0) = -5e^{-2} - (-5) = 5 - \frac{5}{e^2} = 5\left(1 - \frac{1}{e^2}\right)$$

- e. Compute the total amount of snow accumulation from $t = 0$ to $t = 2$.

$$F(2) - F(0) = -5e^{-4} - (-5) = 5\left(1 - \frac{1}{e^4}\right)$$

- f. From the information given so far, can you compute $m(2)$?

Nope.

- g. Suppose $m(0) = 9$. Compute $m(1)$ and $m(2)$.

$$m(1) = 9 + 5\left(1 - e^{-2}\right)$$

$$m(2) = 9 + 5\left(1 - e^{-4}\right)$$

5. $\leftarrow A$ An airplane is descending. Its rate of change of height is $r(t) = -4t + \frac{t^2}{10}$ meters per second.

a. if $A(t)$ is the altitude of the airplane in meters, how are $A(t)$ and $r(t)$ related?

$$A'(t) = r(t)$$

b. What physical quantity does $\int_1^3 r(t) dt$ represent?

The net change in altitude from the first to the third second.

c. Compute $A(3) - A(1)$.

$$\begin{aligned} A(3) - A(1) &= \int_1^3 r(t) dt = \int_1^3 -4t + \frac{t^2}{10} dt \\ &= -2t^2 + \frac{1}{30} t^3 \Big|_1^3 \\ &= \left(-2 \cdot 3^2 + \frac{3^3}{30} \right) - \left(-2 + \frac{1}{30} \right) = -18 + 2 + \frac{27}{30} - \frac{1}{30} = -16 + \frac{26}{30} \\ &\approx -15.13 \text{ m} \end{aligned}$$

6. Gravel is being added to a pile at a rate of $1 + t^2$ tons per minute for $0 \leq t \leq 10$ minutes. If $G(t)$ is the amount of gravel (in tons) in the pile at time t , compute $G(10) - G(0)$.

$$\begin{aligned} G(10) - G(0) &= \int_0^{10} (1 + t^2) dt = t + \frac{1}{3} t^3 \Big|_0^{10} = \left(10 + \frac{10^3}{3} \right) - (0) \\ &= 10 + \frac{1000}{3} \approx 343.3 \text{ tons} \end{aligned}$$