1. Compute $\int e^{4x-9}\,dx$ $= \frac{1}{4}\int e^{u}\,du = \frac{1}{4}e^{u} + C$

$u = 4x-9$

$= \frac{1}{4}e^{4x-9} + C$

$du = 4\,dx$

2. Compute $\int x\sin(x^2+1)\,dx$ $= \frac{1}{2}\int \sin u\,du$

$u = x^2+1$

$= -\frac{1}{2}\cos u + C$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= -\frac{1}{2}\cos(x^2+1) + C$

3. Compute $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx.$ $= \int x^{-1/2}e^{x^{1/2}}\,dx = 2\int e^{u}\,du$

$u = x^{1/2}$

$= 2e^{u} + C$

$du = \frac{1}{2}x^{-1/2}\,dx$

$= 2e^{x^{1/2}} + C$

$2\,du = x^{-1/2}\,dx$

4. Compute $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx.$ $= 2e^{x^{1/2}}\Big|_1^4$

$= 2\left[e^{2} - e^{1}\right] = 2(e^{2}-e)$

5. Compute $\int \frac{\arctan(x)}{1+x^2}\,dx$ $= \int u\,du = \frac{1}{2}u^2 + C$

$u = \arctan x$ $= \frac{1}{2}(\arctan x)^2 + C$

$du = \frac{1}{1+x^2}\,dx$

6. Compute $\int \frac{x^3}{\sqrt{1-x^4}}\,dx$ $= \int x^3(1-x^4)^{-1/2}\,dx = -\frac{1}{4}\int u^{-1/2}\,du$

$u = 1-x^4$

$du = -4x^3\,dx$

$-\frac{1}{4}\,du = x^3\,dx$

$= -\frac{1}{4}\cdot u^{1/2}\cdot 2 + C$

$= -\frac{1}{2}(1-x^4)^{1/2} + C$

7. Compute $\int \frac{x}{\sqrt{1-x^4}}\,dx$. This is harder. The choice of a good substitution can be an art. Try $u = x^2$.

$u = x^2$

$du = 2x\,dx$

$= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\arcsin u + C$

$= \frac{1}{2}\arcsin(x^2) + C$