1. Compute $\int \frac{\sec^2(x)}{\tan(x)}\,dx$ $= \int \frac{du}{u} = \ln|u| + C = \ln|\tan x| + C$

let $u = \tan x$

$du = \sec^2 x\,dx$

2. Compute $\int \sec^2(x)\tan(x)\,dx$ $= \int u\,du = \frac{1}{2}u^2 + C$

let $u = \tan x$

$du = \sec^2 x\,dx$

$= \frac{1}{2}(\tan x)^2 + C$

3. Compute $\int \frac{\sin(\theta)}{1+\cos(\theta)}\,d\theta$ $= -\int \frac{dy}{u} = -\ln|u| + C$

let $u = 1 + \cos\theta$

$du = -\sin\theta\,d\theta$

$= -\ln|1+\cos\theta| + C$

4. Compute $\int \frac{1}{x\ln(x)}\,dx$ $= \int \frac{1}{\ln x}\cdot\frac{dx}{x} = \int \frac{1}{u}\,du = \ln|u| + C$

let $u = \ln x$

$du = \frac{1}{x}\,dx$ $= \ln|\ln x| + C$

5. Compute $\int \frac{\sin(4/x)}{x^2}\,dx$ $= \int x^{-2}\sin(4x^{-1})\,dx = -\frac{1}{4}\int \sin(u)\,du$

$u = 4x^{-1}$

$du = -4x^{-2}\,dx$ $= \frac{1}{4}\cos u + C$

$-\frac{1}{4}\,du = x^{-2}\,dx$ $= \frac{1}{4}\cos(4x^{-1}) + C$

6. Compute $\int \frac{e^x}{e^x - 3}\,dx$ $= \int \frac{du}{u} = \ln|u| + C$

let $u = e^x - 3$ $= \ln|e^x - 3| + C$

$du = e^x\,dx$

7. Compute $\int \frac{1}{9+x^2}\,dx$ $= \int \frac{1}{9} \cdot \frac{1}{\left(1+\left(\frac{x}{3}\right)^2\right)} \cdot dx$

Let $u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$3du = dx$

$= \frac{1}{9}\int \frac{3\,du}{1+u^2} = \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$

8. Compute $\int \sqrt{x}(x^4+x)\,dx$ $= \int \left(x^{9/2} + x^{3/2}\right)dx$

$\frac{1}{2}+\frac{B}{2}$

$= \frac{2}{11}x^{11/2} + \frac{2}{5}x^{5/2} + C$

9. Compute $\int \cos(x)\sin(\sin(x))\,dx$ $= \int \sin u\,du$

Let $u = \sin x$

$du = \cos x\,dx$

$= -\cos u + C$

$= -\cos(\sin x) + C$

10. Compute $\frac{d}{dx}[x\ln(x) - x]$. Then compute $\int s^2 \ln(s^3)\, ds$

$\frac{d}{dx}[x\ln x - x] = 1\cdot\ln x + x\cdot\frac{1}{x} - 1 = \ln x$

that is $\int \ln x\, dx = x\ln x - x + C$

let $u = s^3$

$du = 3s^2 ds$

$\frac{1}{3}du = s^2 ds$

$$\int s^2\ln(s^3)ds = \frac{1}{3}\int \ln u\, du$$

$$= \frac{1}{3}[u\ln u - u] + C$$

$$= \frac{1}{3}[s^3\ln|s^3| - s^3] + C$$

11. Compute $\int x\sqrt{x-1}\, dx$

let $u = x-1$ or $x = u+1$

$du = dx$

$$= \int (u+1)\sqrt{u}\, du$$

$$= \int u^{3/2} + u^{1/2}\, du$$

$$= \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$$

12. Compute $\int_1^3 \frac{(\ln(x))^3}{x}\, dx$

let $u = \ln x$

$du = \frac{1}{x}dx$

when $x = 1$, $u = \ln 1 = 0$

$x = 3$, $u = \ln 3$

$$= \int_0^{\ln 3} u^3\, du$$

$$= \frac{1}{4}u^4\Big]_0^{\ln 3}$$

$$= \frac{1}{4}(\ln 3)^4$$