1. State the definition of the derivative of a function $f(x)$ at $x = a$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \quad \left(\text{or: } = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}\right)$$

2. Let $f(x) = 5x^2 - 3x$.

(a) Use the definition to find the derivative of $f(x)$.

$$f'(x) = \lim_{h\to 0} \frac{5(x+h)^2 - 3(x+h) - (5x^2 - 3x)}{h}$$

$$= \lim_{h\to 0} \frac{5(\cancel{x^2} + 2xh + h^2) - \cancel{3x} - 3h - \cancel{5x^2} + \cancel{3x}}{h}$$

$$= \lim_{h\to 0} \frac{10xh + h^2 - 3h}{h} = \lim_{h\to 0} \frac{\cancel{h}(10x + h - 3)}{\cancel{h}}$$

$$= \lim_{h\to 0} 10x + h - 3 = 10x + 0 - 3 = 10x - 3$$

(b) Find the slope of the tangent line to $f(x)$ when $x = -3$.

$$m = f'(-3) = 10(-3) - 3 = -33$$

(c) Write the equation of the line tangent to $f(x)$ when $x = -3$.

$$y = f(-3) = 5(-3)^2 - 3(-3) = 45 + 9 = 54$$

$$y - 54 = -33(x - (-3))$$

3. Suppose N represents the number of people in the United States who travel by car to another state for a vacation this Memorial Day weekend when the average price of gasoline is p dollars per gallon.

(a) What are the units of dN/dp? people per dollar $\left(\frac{\text{people}}{\text{dollar}}\right)$

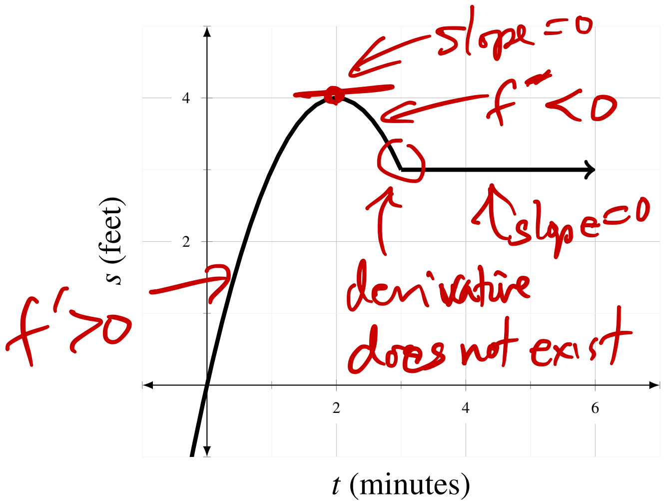
(b) In the context of the problem, write a sentence interpreting $\frac{dN}{dp}$.

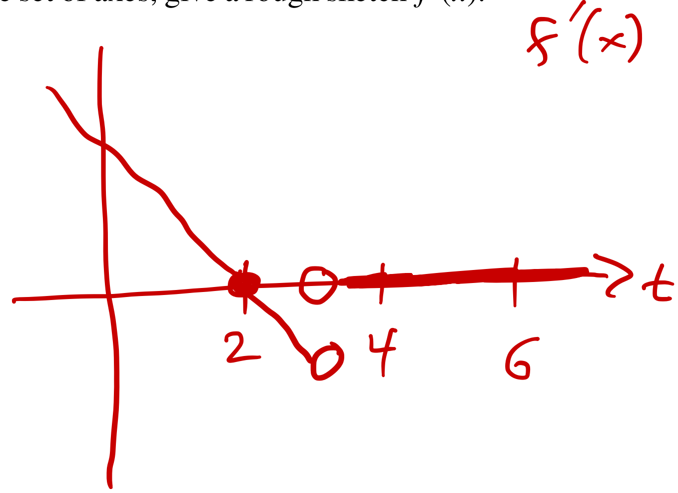
$\frac{dN}{dp}$ is the rate at which the number of out-of-state vacationers changes as the price of gas increases

(c) Would you expect dN/dp to be positive or negative? Explain your answer.

$\frac{dN}{dp} < 0$ because people drive less if gas costs more

4. The graph of $f(x)$ is sketched below. On a separate set of axes, give a rough sketch $f'(x)$.





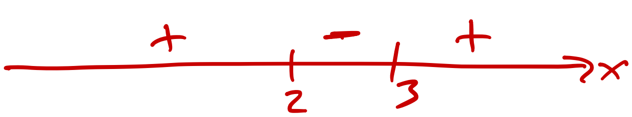
5. Find the domain of each function.

(a) $f(x) = \sqrt{x^2 - x - 6}$

$x^2 - x - 6 = (x-3)(x-2) > 0$

$\iff$ $x \le -2$ or $x \ge 3$

$\iff$ $(-\infty, -2] \cup [3, \infty)$



(b) $g(t) = \ln(t + 6)$

$t + 6 > 0$

$t > -6$

or $(-6, \infty)$

6. State the definition of "The function $f(x)$ is continuous at $x = a$".

① $x=a$ is in domain of f

② $\lim_{x\to a} f(x)$ exists

③ $f(a) = \lim_{x\to a} f(x)$

[most important]

7. Suppose

$$f(x) = \begin{cases} -\frac{2}{x} & x < 2 \\ \frac{x}{x-3} & x \geq 2 \end{cases}$$

Is $f(x)$ continuous at $x = 0$? At $x = 2$? Justify your answers using the definition of continuity.

$x=0$: $f(x) = \frac{-2}{x}$ near zero $\therefore$ f is not continuous because $x=0$ is not in domain f

$x=2$: $f(2) = \frac{2}{2-3} = -2$, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{-2}{x} = \frac{-2}{2} = -1$ $\therefore$ f is not continuous at $x=2$

8. Find the limit or show that it does not exist. *Make sure you are writing your mathematics correctly and clearly.*

(a) $\lim_{x\to\infty} \frac{10^x - 1}{3 - 10^x} = \lim_{x\to\infty} \frac{10^x - 1}{3 - 10^x} \cdot \frac{\frac{1}{10^x}}{\frac{1}{10^x}} = \lim_{x\to\infty} \frac{1 - 10^{-x}}{3 \cdot 10^{-x} - 1}$

$= \frac{1-0}{0-1} = -1$

(b) $\lim_{x\to\infty} \frac{\sqrt[3]{8x^3 + 1}}{2 - 5x} = \lim_{x\to\infty} \frac{\sqrt[3]{8x^3+1}}{2-5x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{\sqrt[3]{8\frac{x^3}{x^3} + \frac{1}{x^3}}}{\frac{2}{x} - 5} = \lim_{x\to\infty} \frac{\sqrt[3]{8 + \frac{1}{x^3}}}{\frac{2}{x} - 5}$

$= \frac{\sqrt[3]{8+0}}{0-5} = \frac{2}{-5} = -\frac{2}{5}$

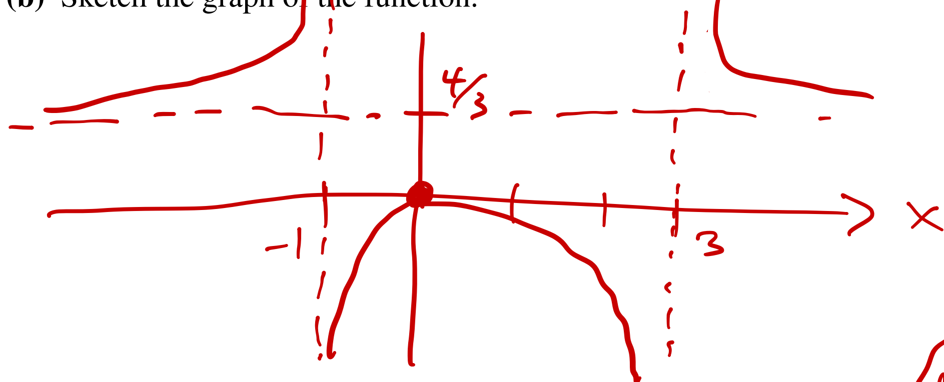
9. Consider a function with vertical asymptotes at $x = -1$ and $x = 3$ and a horizontal asymptote at $y = 4/3$.

(a) Write a formula for such a function.

$$f(x) = \frac{\frac{4}{3}x^2}{(x+1)(x-3)}$$

or: $g(x) = \frac{1}{(x+1)(x+3)} + \frac{4}{3}$ ... there are many correct answers!

(b) Sketch the graph of the function.



10. Solve for x.

(a) $e^{x-3} + 2 = 6$

$e^{x-3} = 4$

$x - 3 = \ln 4$

$x = \ln 4 + 3$

(b) $\ln(x + 5) - 3 = 7$

$\ln(x+5) = 10$

$x + 5 = e^{10}$

$x = e^{10} - 5$

(c) $\ln x + \ln(x - 1) = 0$

domain: $(1, \infty)$

$\ln(x(x-1)) = 0$

$x(x-1) = e^0 = 1$

$x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

$x = \frac{1+\sqrt{5}}{2}$

$\left(\frac{1-\sqrt{5}}{2}\right.$ is not in domain$\left.\right)$

(d) $\cos(8x) = 0$

$8x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$= \frac{\pi}{2} + k\pi$

$x = \frac{\pi}{16} + k\frac{\pi}{8}$

[k is an integer]

11. Use the Intermediate Value Theorem to show $\ln x = x - 5$ has a solution. (Hint: Show there is a solution in the interval $[1, e^5]$.)

$f(x) = \ln x - x + 5$ is continuous on $(0, \infty)$

$f(1) = \ln 1 - 1 + 5 = 4$

$f(e^5) = \ln(e^5) - e^5 + 5 = 5 + 5 - e^5 = 10 - e^5 < 0$

by IVT there is c so that $f(c) = 0$ $\quad (e^5 > 2^5 = 32)$

12. Sketch each of the functions below. Label all x- and y-intercepts and asymptotes. State, in interval notation, the domain and range of each function next to its graph.

(a) $y = 6 - x^4$

(b) $y = \sin(2x)$

(c) $y = \tan x$

(d) $y = \tan^{-1} x$

(e) $y = e^{x-1} + 2$

(f) $y = \ln x$

(g) $y = -2/(x + 3)$

(h) $y = \sqrt{x + 5}$