Circle your Instructor:
Faudree, Williams, Zirbes

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Math 251 Fall 2017 **Derivative Proficiency, October 25th**

Name: Solutions

This is a 30 minute quiz. There are 15 problems. Books, notes, calculators or any other aids are prohibited. Calculators and notes are not allowed. **Your answers should be simplified unless otherwise stated.** They should begin $y' =$ or $f'(x) =$ or $dy/dx =$, etc. There is no partial credit. If you have any questions, please raise your hand.

Circle your final answer.

For each function below, find the derivative.

1. $y = \frac{\ln x}{x^3} = x^{-3} \ln x$

$y' = -3x^{-4} \ln x + x^{-3} \cdot \frac{1}{x} = -3x^{-4} \ln x + x^{-4}$

$\boxed{y' = x^{-4}(-3\ln x + 1)}$

2. $g(x) = \sqrt{3x} - x^{\pi-1} + \frac{x}{3} = \sqrt{3}\, x^{1/2} - x^{\pi-1} + \frac{1}{3}x$

$\boxed{g'(x) = \frac{\sqrt{3}}{2} x^{-1/2} - x^{\pi-2} + \frac{1}{3}}$

3. $f(x) = 6^{x^2} + \cot(x)$

$f'(x) = (\ln 6) \cdot 6^{x^2} \cdot 2x + (-\csc^2 x)$

$\boxed{f'(x) = (2\ln 6)x\, 6^{x^2} - \csc^2 x}$

4. $y = \frac{-\pi}{(x^2+7x)^3} = -\pi(x^2+7x)^{-3}$

$y' = -\pi(-3)(x^2+7x)^{-4} \cdot (2x+7)$

$\boxed{y' = 3\pi(2x+7)(x^2+7x)^{-4}}$

5. $h(x) = \frac{\cos 3x}{3+x-x^2}$ [You don't need to simplify.]

$\boxed{h'(x) = \frac{(3+x-x^2)(-3\sin 3x) - (\cos(3x))(1-2x)}{(3+x-x^2)^2}}$

6. $y = \sqrt{4x^2-25} = (4x^2-25)^{1/2}$

$y' = \frac{1}{2}(4x^2-25)^{-1/2}(8x)$

$\boxed{y' = 4x(4x^2-25)^{-1/2}}$

7. $F(t) = (t^{-1}+8)e^{-1/t} = (t^{-1}+8)e^{-t^{-1}}$

$F'(t) = (t^{-1}+8) \cdot e^{-t^{-1}} \cdot (-1)(-1)t^{-2} + (-1 \cdot t^{-2}) \cdot e^{-t^{-1}}$

$= t^{-2}(t^{-1}+8)e^{-1/t} - t^{-2} \cdot e^{-1/t}$

$F'(t) = t^{-2}e^{-1/t}(t^{-1}+8-1)$

$= t^{-2}e^{-1/t}(t^{-1}+7)$ or $= \frac{e^{-1/t}(1+7t)}{t^3}$

aside

$t^{-1}+7 = \frac{1}{t}+7 = \frac{1+7t}{t}$

8. $f(x) = \arctan(e^{4x})$

$$f'(x) = \frac{1}{1+(e^{4x})^2} \cdot e^{4x} \cdot 4$$

$$\boxed{f'(x) = \frac{4e^{4x}}{1+e^{8x}}}$$

9. $y = \ln(3x+1)\arccos(x^2)$

$$y' = \ln(3x+1) \cdot \frac{-1}{\sqrt{1-x^4}} \cdot 2x + \frac{3}{3x+1} \cdot \arccos(x^2)$$

$$\boxed{y' = \frac{-2x\ln(3x+1)}{\sqrt{1-x^4}} + \frac{3\arccos(x^2)}{3x+1}}$$

10. $G(x) = \ln\left(\frac{(x+2)^2}{8x}\right) = 2\ln(x+2) - \ln(8x) = 2\ln(x+2) - \ln 8 - \ln x$

$$G'(x) = 2 \cdot \frac{1}{x+2} \cdot 1 - 0 - \frac{1}{x}$$

$$\boxed{G'(x) = \frac{2}{x+2} - \frac{1}{x}}$$

11. $z = y^2(\sqrt{y} - 18\sqrt[3]{y}) = y^2\left(y^{\frac{1}{2}} - 18y^{\frac{1}{3}}\right) = y^{5/2} - 18y^{7/3}$

$$z' = \frac{5}{2}y^{3/2} - \frac{18 \cdot 7}{3}y^{4/3}$$

$$\boxed{z' = \frac{5}{2}y^{3/2} - 42y^{4/3}}$$

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12. $h(x) = \frac{-4}{(\sec(4x))^{5/4}} = -4(\sec(4x))^{-5/4}$

$$h'(x) = -4\left(-\frac{5}{4}\right)(\sec(4x))^{-9/4} \cdot \sec(4x)\tan(4x) \cdot 4$$

$$\boxed{h'(x) = 20\tan(4x) \cdot \sec(4x)^{-5/4}}$$

13. (You do not need to simplify, in this case.) $H(x) = x^2 e^x (\arccos x)$

$$\boxed{H'(x) = 2x \cdot e^x \arccos x + x^2\left[e^x \cdot \arccos x + e^x \cdot \frac{-1}{\sqrt{1-x^2}}\right]}$$

14. $g(x) = (\sin^3(x) + x)^4$

$$\boxed{g'(x) = 4(\sin^3 x + x)^3 \cdot [3\sin^2 x \cdot \cos x + 1]}$$

15. Find dH/dx for $H = (ax+b)(cx)^3$ where a, b, and c are fixed constants.

$$H = (ax+b) \cdot c^3 \cdot x^3 = c^3(ax+b)x^3$$

$$\frac{dH}{dx} = c^3[a \cdot x^3 + (ax+b) \cdot 3x^2]$$

$$\boxed{\frac{dH}{dx} = c^3x^2(ax+3ax+3b)} = \boxed{c^3x^2(4ax+3b)}$$