

Name: _____

SOLUTIONS

_____/12

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- Circle your final answer.

Compute the derivatives of the following functions.

1. $f(x) = \sqrt{x} + \sqrt{6} - \frac{e^x}{3}$

$$f'(x) = \frac{1}{2}x^{-1/2} - \frac{e^x}{3}$$

2. $f(t) = \frac{1+5t-t^{4/3}}{t} = t^{-1} + 5 - t^{1/3}$

$$f'(t) = -t^{-2} - \frac{1}{3}t^{-2/3}$$

3. $y = x^2 \sec(x)$

$$\frac{dy}{dx} = 2x \sec(x) + x^2 \sec(x) \tan(x)$$

4. $y = e^{-ax} \cos(bx)$, where a and b are fixed constants

$$y' = -a e^{-ax} \cos(bx) + e^{-ax} (-\sin(bx)) b$$

5. $f(x) = \arctan(\sin(5x))$

$$f'(x) = \frac{1}{1 + (\sin(5x))^2} \cdot \cos(5x) \cdot 5$$

6. $f(x) = \frac{\cos(x)}{\sin(x)} = \cot(x)$

$$f'(x) = -\csc^2(x)$$

7. $y = \frac{xe^x}{1-x}$

$$\frac{dy}{dx} = \frac{(1 \cdot e^x + xe^x)(1-x) - xe^x(-1)}{(1-x)^2}$$

8. $y = \tan(x + \sqrt{x})$

$$\frac{dy}{dx} = \sec^2(x + \sqrt{x}) \left(1 + \frac{1}{2}x^{-1/2}\right)$$

9. $f(x) = \sqrt{x} \ln(x) \sin(\pi x)$

$$\begin{aligned} f'(x) &= \frac{1}{2}x^{-1/2} \ln(x) \sin(\pi x) \\ &\quad + \sqrt{x} \frac{1}{x} \sin(\pi x) \\ &\quad + \sqrt{x} \ln(x) \cos(\pi x) \pi \end{aligned}$$

10. $f(x) = x + \sqrt{x^2 + 1}$

$$f'(x) = 1 + \frac{1}{2}(x^2 + 1)^{-1/2}(2x)$$

11. $g(t) = \frac{\ln 3}{1-t^2} = \ln(3)(1-t^2)^{-1}$

$$g'(t) = -\ln(3)(1-t^2)^{-2}(-2t)$$

12. Compute dy/dx if $2xy^2 - x^3 + y^5 = 0$. You must solve for dy/dx .

$$2y^2 + 2x \cdot 2yy' - 3x^2 + 5y^4 y' = 0$$

$$y'(4xy + 5y^4) = -2y^2 + 3x^2$$

$$\frac{dy}{dx} = \frac{-2y^2 + 3x^2}{4xy + 5y^4}$$