

Name: Solutions

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- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- Be sure to include constants of integration where appropriate.
- You do **not** need to simplify your expressions.
- Box your final answer.

Evaluate the integrals.

$$1. \int \left(\frac{3}{x^2} - \frac{x}{5} + \frac{\sqrt{2}}{2} \right) dx = \int 3x^{-2} - \frac{1}{5}x + \frac{\sqrt{2}}{2} dx$$

$$= -3x^{-1} - \frac{1}{10}x^2 + \frac{\sqrt{2}}{2}x + C$$

$$2. \int_0^{\pi/6} (e^t - \sin(t)) dt = e^t + \cos t \Big|_0^{\pi/6} = e^{\pi/6} + \cos\left(\frac{\pi}{6}\right) - (e^0 + \cos 0)$$

$$= e^{\pi/6} + \frac{\sqrt{3}}{2} - (1+1)$$

$$= e^{\pi/6} + \frac{\sqrt{3}}{2} - 2$$

$$3. \int \pi^2 dx = \pi^2 x + C$$

$$4. \int \sec(\theta/2) \tan(\theta/2) d\theta = 2 \sec(\theta/2) + C$$

$$5. \int \frac{1 + \sqrt{x}}{x^3} dx = \int (x^{-3} + x^{-5/2}) dx$$

$$= \boxed{-\frac{1}{2} x^{-2} - \frac{2}{3} x^{-3/2} + C}$$

$$6. \int (\sec v)^2 (1 + \tan v)^3 dv = \int u^3 du = \frac{1}{4} u^4 + C$$

$$\text{let } u = 1 + \tan v$$

$$du = \sec^2 v dv$$

$$= \frac{1}{4} (1 + \tan v)^4 + C$$

$$7. \int_1^2 \frac{\ln x}{2x} dx = \frac{1}{2} \int_0^{\ln 2} u du = \frac{1}{4} u^2 \Big|_0^{\ln 2} = \frac{1}{4} (\ln 2)^2$$

$$\text{let } u = \ln x \\ du = \frac{1}{x} dx$$

$$\text{when } x=1, u = \ln 1 = 0$$

$$x=2, u = \ln 2$$

$$8. \int e^{2x} \cos(3e^{2x}) dx = \frac{1}{6} \int \cos u du = \frac{1}{6} \sin u + C$$

$$\text{let } u = 3e^{2x}$$

$$du = 6e^{2x} dx$$

$$\frac{1}{6} du = e^{2x} dx$$

$$= \frac{1}{6} \sin(3e^{2x}) + C$$

$$9. \int \frac{17e^{\sqrt{x}}}{\sqrt{x}} dx = 17 \cdot 2 \cdot \int e^u du = 34e^u + C$$

$$\text{let } u = x^{\frac{1}{2}}$$

$$du = \frac{1}{2} x^{-\frac{1}{2}} dx$$

$$2 du = \frac{dx}{\sqrt{x}}$$

$$= 34e^{\sqrt{x}} + C$$

$$\begin{aligned}
 10. \int x\sqrt{x-2} \, dx &= \int (u+2)(u^{\frac{1}{2}}) \, du \\
 \text{let } u &= x-2 \\
 du &= dx \\
 u+2 &= x \\
 &= \int (u^{\frac{3}{2}} + 2u^{\frac{1}{2}}) \, du \\
 &= \frac{2}{5} u^{\frac{5}{2}} + 2\left(\frac{2}{3}\right) u^{\frac{3}{2}} + C = \frac{2}{5} (x-2)^{\frac{5}{2}} + \frac{4}{3} (x-2)^{\frac{3}{2}} + C
 \end{aligned}$$

$$11. \int x + \frac{x^2}{x^3+1} \, dx = \frac{1}{2}x^2 + \frac{1}{3} \ln|x^3+1| + C$$

$$12. \int \left(e^{-x} + \frac{5}{\sqrt{1-x^2}} \right) dx = -e^{-x} + 5 \arcsin(x) + C$$