Name: Solutions ______ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- Be sure to include constants of integration where appropriate.
- You do **not** need to simplify your expressions.
- Box your final answer.

Evaluate the integrals.

1. $\int \left(\frac{2}{x^2} - \frac{x}{4} + \frac{\sqrt{3}}{3} \right) \, dx = \int 2x^{-2} - \frac{1}{4}x + \frac{\sqrt{3}}{3}x \, dx$

$= \boxed{-2x^{-1} - \frac{1}{8}x^2 + \frac{\sqrt{3}}{3}x + C}$

2. $\int_0^{\pi/3} \left(e^t - \sin(t)\right) \, dt = e^t + \cos t \Big]_0^{\pi/3} = \left(e^{\pi/3} + \cos(\pi/3)\right) - \left(e^0 + \cos 0\right)$

$1 + 1$

$= e^{\pi/3} - 2 + \frac{1}{2}$

$= \boxed{e^{\pi/3} - \frac{3}{2}}$

3. $\int \sec(\theta/5)\tan(\theta/5) \, d\theta = \boxed{5\sec(\theta/5) + C}$

$4 = \frac{8}{2}$

4. $\int \frac{1+\sqrt{x}}{x^4}\,dx$ $= \int \left(x^{-4} + x^{-7/2}\right) dx$

$$= \boxed{-\frac{1}{3}x^{-3} - \frac{2}{5}x^{-5/2} + C}$$

5. $\int \pi^2\,dx$ $= \boxed{\pi^2 x + C}$

6. $\int (\sec v)^2 (1+\tan v)^3\,dv$ $= \int u^3\,dv = \frac{1}{4}u^4 + C$

let $u = 1+\tan v$

$du = \sec^2 v\,dv$

$$= \boxed{\frac{1}{4}(1+\tan v)^4 + C}$$

7. $\int \frac{11e^{\sqrt{x}}}{\sqrt{x}}\,dx$ $= 11 \cdot 2 \int e^u\,du = 22e^u + c$

$= \boxed{22e^{\sqrt{x}} + c}$

let $u = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2\,du = x^{-1/2}\,dx$

8. $\int_1^2 \frac{\ln x}{3x}\,dx$ $= \frac{1}{3}\int_0^{\ln 2} u\,du = \frac{1}{6}u^2\Big|_0^{\ln 2}$

$= \boxed{\frac{1}{6}(\ln 2)^2}$

let $u = \ln x$

$du = \frac{1}{x}\,dx$

$x=1,\ u=\ln 1 = 0$

$x=2,\ u=\ln 2$

9. $\int e^{2x}\cos(3e^{2x})\,dx$ $= \frac{1}{6}\int \cos u\,du$

let $u = 3e^{2x}$

$du = 6e^{2x}\,dx$

$\frac{1}{6}\,du = e^{2x}\,dx$

$= \frac{1}{6}\sin u + c$

$= \frac{1}{6}\sin(3e^{2x}) + c$

10. $\int x+\frac{x^2}{x^3+1}\,dx$ $= \int x\,dx + \int \frac{x^2\,dx}{x^3+1}$

$= \boxed{\frac{1}{2}x^2 + \frac{1}{3}\ln\left|x^3+1\right| + C}$

11. $\int x\sqrt{x-1}\,dx$ $= \int (u+1)u^{1/2}\,du = \int u^{3/2}+u^{1/2}\,du$

let $u=x-1$
$du=dx$
$u+1=x$

$= \frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}+C$

$= \boxed{\frac{2}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+C}$

12. $\int \left(\frac{8}{\sqrt{1-x^2}}+e^{-x}\right)\,dx$ $= \boxed{8\arcsin x - e^{-x}+C}$