Name: Solutions Class (circle): Sync. Online

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- A passing score is 10/12.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- Do **not** simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- $\boxed{\textbf{Box}}$ **your final answer.**

Compute the derivatives of the following functions.

1. $f(x) = 15x^2 - \dfrac{3}{x} + x\sqrt{2} - \dfrac{15}{2}$ $= 15x^2 - 3x^{-1} + (\sqrt{2})x - 15/2$

$f'(x) = 15(2x) + 3x^{-2} + \sqrt{2}$ ← This is fine!

$$\boxed{f'(x) = 30x + \frac{3}{x^2} + \sqrt{2}}$$

2. $g(t) = \cos(5t)\left(e^{2t} + 3\right)$

$$\boxed{g'(t) = \cos(5t)\left(e^{2t}(2)\right) + \left(e^{2t} + 3\right)\left(-\sin(5t)(5)\right)}$$

3. $y = \ln(\sec(5x))$

$$\boxed{y' = \frac{1}{\sec(5x)} \cdot \sec(5x)\tan(5x) \cdot 5}$$

$= 5\tan(5x)$ but you did not have to simplify

4. $h(x) = \dfrac{\tan(x)}{x+\ln(x)} = \tan(x)\,(x+\ln(x))^{-1}$

$$\boxed{h'(x) = \frac{(x+\ln(x))\cdot(\sec(x))^2 - \tan(x)\left(1+\frac{1}{x}\right)}{(x+\ln(x))^2}}$$

or

$$h'(x) = \tan(x)\left(-(x+\ln(x))^{-2}\right)\left(1+\tfrac{1}{x}\right) + (x+\ln(x))^{-1}\left((\sec(x))^2\right)$$

5. $D(r) = \dfrac{r^2-5r+\pi}{17r^4} = \frac{1}{17}\left(r^{-2} - 5r^{-3} + \pi r^{-4}\right) = (r^2-5r+\pi)(17r^4)^{-1}$

$$\boxed{D'(r) = \frac{1}{17}\left(-2r^{-3} - 5(-3)r^{-4} + \pi(-4)r^{-5}\right)}$$

or $D'(r) = (r^2-5r+\pi)\left(-1(17r^4)^{-2}(17\cdot 4r^3)\right) + (17r^4)^{-1}(2r-5)$

or $D'(r) = \dfrac{17r^4(2r-5) - (r^2-5r+\pi)(17\cdot 4r^3)}{(17r^4)^2}$

6. $r(\theta) = 8\pi - (\sin(b\theta))^2$, where b is a fixed constant

$$\boxed{r'(\theta) = 0 - 2(\sin(b\theta))(\cos(b\theta))\cdot b}$$

7. $h(s) = \sqrt{\dfrac{s^2 - 3s + 7}{6}}$ $= \frac{1}{\sqrt{6}} \left(s^2 - 3s + 7 \right)^{1/2}$

$$\boxed{h'(s) = \frac{1}{\sqrt{6}} \left(\frac{1}{2} \right) \left(s^2 - 3s + 7 \right)^{-1/2} \left(2s - 3 \right)}$$

8. $f(x) = \ln(3x) \left(\sec(x) e^{7x} \right)$

$$\boxed{f'(x) = \ln(3x) \left[\sec(x) \cdot e^{7x} \cdot 7 + e^{7x} \sec(x) \tan(x) \right] + \left(\sec(x)\, e^{7x} \right) \cdot \frac{1}{3x} (3)}$$

9. $y = \arcsin\left(5x^3 - 4\right)$

$$\boxed{y' = \frac{1}{\sqrt{1 - (5x^3 - 4)^2}} \left(15x^2 \right)}$$

10. $s(t) = e^3 - \ln(4) + \dfrac{t^2}{\sqrt{5}}$

$$\boxed{s'(t) = \frac{1}{\sqrt{5}}(2t)}$$

← Note e^3 and $\ln(4)$ are both constants.

$e^3 \approx 20.085$

$\ln(4) \approx 1.386$

11. $g(\theta) = \tan(\theta)\cos(\theta) = \dfrac{\sin\theta}{\cos\theta} \cdot \cos\theta = \sin\theta$

$$\boxed{g'(\theta) = \cos\theta}$$

or, $g'(\theta) = \tan\theta\,(-\sin\theta) + \cos(\theta)\,(\sec\theta)^2$

$= -\dfrac{\sin\theta}{\cos\theta}(\sin\theta) + \cos\theta \cdot \dfrac{1}{(\cos\theta)^2} = \dfrac{-\sin^2\theta}{\cos\theta} + \dfrac{1}{\cos\theta} = \dfrac{1-\sin^2\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta}{\cos\theta} = \cos\theta.$

12. Compute dy/dx if $y\cos(x) + e^y = xy$. You must solve for dy/dx.

$-y\sin(x) + \cos(x)\dfrac{dy}{dx} + e^y\dfrac{dy}{dx} = x\dfrac{dy}{dx} + y \implies$

$\dfrac{dy}{dx}\left(\cos(x) + e^y - x\right) = y + y\sin(x) \implies$

$$\boxed{\frac{dy}{dx} = \frac{y + y\sin(x)}{\cos(x) + e^y - x}}$$