**Name:** ______________________________ **Class (circle):** **Berman/Sus** **Jurkowski**

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- A passing score is 10/12.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- Be sure to include constants of integration where appropriate.
- You do **not** need to simplify your expressions.
- Box your final answer.

Evaluate the integrals.

1. $\int \left(2x^5 - \sqrt{2}\right)\, dx$

$$= \boxed{\frac{2x^6}{6} - (\sqrt{2})x + C}$$

2. $\int \left(\frac{2+t+\sqrt{t}}{\sqrt{t}}\right)\, dt$

$$= \int 2t^{-1/2} + t^{1/2} + 1 \; dt$$

$$= \boxed{\frac{2t^{1/2}}{1/2} + \frac{t^{3/2}}{3/2} + t + C}$$

3. $\int 2\theta^2 \sin\left(\theta^3\right)\, d\theta$

$$u = \theta^3 \qquad \frac{du}{d\theta} = 3\theta^2 \Rightarrow \frac{du}{3\theta^2} = d\theta$$

$$\int 2\theta^2 \sin(\theta^3)\, d\theta = \int 2\cancel{\theta^2} \cdot \sin(u) \cdot \frac{du}{3\cancel{\theta^2}} = \frac{2}{3}\int \sin(u)\, du$$

$$= -\frac{2}{3}\cos(u) + C = \boxed{-\frac{2}{3}\cos(\theta^3) + C}$$

4. $\int_1^3 \left(x^2 - 4x + 2\right)\, dx$

$$= \frac{x^3}{3} - 4\left(\frac{x^2}{2}\right) + 2x \Big|_1^3$$

$$= \boxed{\left[\frac{3^3}{3} - 2 \cdot 3^2 + 2(3)\right] - \left[\frac{1}{3} - 2 + 2\right]}$$

5. $\int \sin(2t)(\cos(2t))^4\, dt$

$$u = \cos(2t) \Rightarrow \frac{du}{dt} = -\sin(2t) \cdot 2 \Rightarrow$$

$$\frac{du}{-2\sin(2t)} = dt$$

$$= \int \sin(2t) \cdot u^4 \cdot \frac{du}{-2\sin(2t)} = -\frac{1}{2}\int u^4\, du$$

$$= -\frac{1}{2}\frac{u^5}{5} + c = \boxed{\frac{-1}{10}\left(\cos(2t)\right)^5 + c}$$

6. $\int \frac{\cos\left({}^1\!/_t\right)}{t^2}\, dt$

$$u = {}^1\!/_t \qquad du = \frac{-1}{t^2}\, dt$$

$$\int \frac{\cos({}^1\!/_t)}{t^2}\, dt = -\int \cos(u)\, du$$

$$= -\sin(u) + c$$

$$= \boxed{-\sin\left(\frac{1}{t}\right) + c}$$

7. $\int x\sqrt{x+2}\,dx$

$u = x+2 \Rightarrow u-2 = x$ and $du = dx$

so $\int x\sqrt{x+2}\,dx = \int (u-2)\sqrt{u}\,du$

$= \int u^{3/2} - 2u^{1/2}\,du$

$= \boxed{\dfrac{u^{5/2}}{5/2} - \dfrac{2u^{3/2}}{3/2} + c}$

8. $\int \left(e^x + \frac{\sec(x)\tan(x)}{2}\right) dx$

$= \boxed{e^x + \frac{1}{2}\sec(x) + c}$

9. $\int_1^e \frac{(\ln y)^{1/3}}{y}\,dy$

$u = \ln(y) \Rightarrow \frac{du}{dy} = 1/y$

If $y=1$, $u = \ln(1) = 0$, and if $y = e$, $u = \ln(e) = 1$.

So $\int_1^e \frac{\ln(y)^{1/3}}{y}\,dy = \int_{u=0}^{u=1} u^{1/3}\,du = \left.\frac{u^{4/3}}{4/3}\right|_0^1 = \frac{1^{4/3}}{\frac{4}{3}} - \frac{0^{4/3}}{4/3}$

$= \boxed{\frac{3}{4}}$

10. $\int (x+2)(x^2+4x)\,dx$

$= \int x^3 + 2x^2 + 4x^2 + 8x\,dx$

$= \int x^3 + 6x^2 + 8x\,dx$

$= \boxed{\frac{x^4}{4} + \frac{6 \cdot x^3}{3} + \frac{8x^2}{2} + c}$

$= \frac{x^4}{4} + 2x^3 + 4x^2 + c$

Alternately, let $u = x^2+4x$.

Then $\frac{du}{dx} = 2x+4$, so

$\int (x+2)(x^2+4x)\,dx =$

$\int (x+2)(u) \cdot \frac{du}{2(x+2)} =$

$\frac{1}{2}\int u\,du = \frac{1}{2}\frac{u^2}{2} + c$

$= \boxed{\frac{1}{4}(x^2+4x)^2 + c}$

$= \frac{1}{4}(x^4 + 8x^3 + 16x^2) + c$

$= \frac{x^4}{4} + 2x^3 + 4x^2 + c$

11. $\int \left(\frac{2}{2w+3} + \frac{1}{1+w^2}\right) dw$

$u = 2w+3 \Rightarrow \frac{du}{dw} = 2 \Rightarrow \frac{du}{2} = dw$

So $\int \frac{2}{2w+3}\,dw + \int \frac{1}{1+w^2}\,dw$

$= \int \frac{2}{u} \cdot \frac{du}{2} + \int \frac{1}{1+w^2}\,dw$

$= \ln|u| + \arctan(w) + c = \ln|2w+3| + \arctan(w) + c$

12. $\int \left(\sec^2\left(\frac{x}{3}\right) + e^{-x}\right) dx$

$= \int \left(\sec\left(\frac{x}{3}\right)\right)^2 dx + \int e^{-x}\,dx$

let $u = \frac{x}{3} \Rightarrow 3\,du = dx$

and $v = -x \Rightarrow -dv = dx$

$= \int (\sec(u))^2 \cdot 3\,du + \int -e^v\,dv$

$= 3\tan(u) - e^v + c$

$= \boxed{3\tan\left(\frac{x}{3}\right) - e^{-x} + c}$