Name: ______________________ **Instructor (circle):** **Maxwell Jurkowski Sus**

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\displaystyle\int_1^4 \frac{x+1}{\sqrt{x}}\,dx = \int_1^4 (x+1)x^{-1/2}\,dx = \int_1^4 \left(x^{1/2} + x^{-1/2}\right)dx$

$= \left.\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}}\right]_1^4 = \left(\frac{2}{3}(4)^{3/2} + 2(4)^{1/2}\right) - \left(\frac{2}{3}(1)^{3/2} + 2(1)^{1/2}\right)$ ok here

$= \left(\frac{2}{3}(8) + 4\right) - \left(\frac{2}{3} + 2\right) = \frac{16}{3} + 4 - \frac{2}{3} - 2 = 2 + \frac{14}{3} = \boxed{\frac{20}{3}}$

b. $\displaystyle\int_0^{1/2} (4 - \cos(\pi x))\,dx = \left.4x - \frac{1}{\pi}\sin(\pi x)\right]_0^{1/2} = \left(4\left(\frac{1}{2}\right) - \frac{\sin(\pi/2)}{\pi}\right) - \left(4\cdot 0 - \frac{\sin(0)}{\pi}\right)$

ok here

$= \left(2 - \frac{1}{\pi}\right) - (0 - 0) = \boxed{2 - \frac{1}{\pi}}$

c. $\displaystyle\int (x+3)(2x+1)\,dx = \int \left(2x^2 + 7x + 3\right)dx = \boxed{\frac{2}{3}x^3 + \frac{7}{2}x^2 + 3x + C}$

expand

$2x^2 + x + 6x + 3$

$= 2x^2 + 7x + 3$

d. $\int xe^{3x^2}\,dx$ $= \frac{1}{6}\int e^u\,du = \frac{1}{6}e^u + C$

let $u = 3x^2$

$du = 6x\,dx$

$\frac{1}{6}du = x\,dx$

$= \boxed{\frac{1}{6}e^{3x^2} + C}$

e. $\int \frac{\cos(x)+1}{\sin(x)+x}\,dx$ $= \int \frac{du}{u} = \ln|u| + C$

let $u = \sin x + x$

$du = (\cos x + 1)\,dx$

$= \boxed{\ln|\sin x + x| + C}$

f. $\int \frac{e^x}{(5+e^x)^4}\,dx$ $= \int e^x(5+e^x)^{-4}\,dx = \int u^{-4}\,du = -\frac{1}{3}u^{-3} + C$

let $u = 5 + e^x$

$du = e^x\,dx$

$= \boxed{-\frac{1}{3}(5+e^x)^{-3} + C}$

g. $\int \sec(1-2x)\tan(1-2x)\,dx = -\frac{1}{2}\int \sec(u)\tan(u)\,du$

let $u = 1-2x$

$du = -2\,dx$

$-\frac{1}{2}du = dx$

$= -\frac{1}{2}\sec(u) + C$

$= \boxed{-\frac{1}{2}\sec(1-2x) + C}$

h. $\int \frac{6}{1+x^2}\,dx = \boxed{6\arctan(x) + C}$

i. $\int x(x+1)^{10}\,dx = \int (u-1)u^{10}\,du = \int (u^{11} - u^{10})\,du = \frac{1}{12}u^{12} - \frac{1}{11}u^{11} + C$

let $u = x+1$

$du = dx$

$x = u-1$

$= \boxed{\frac{1}{12}(x+1)^{12} - \frac{1}{11}(x+1)^{11} + C}$

j. $\int \sqrt{2}\sec^2(x)\,dx$ $= \boxed{\sqrt{2}\tan(x) + C}$

k. $\int \frac{1}{x} + \frac{\ln(x)}{x}\,dx$ $= \int \frac{1}{x}dx + \int \frac{\ln x}{x}dx = \ln|x| + \int \frac{\ln x}{x}dx = \ln|x| + \int u\,du$

let $u = \ln x$

$du = \frac{1}{x}dx$

note: In this case, absolute value bars unnecessary.

$= \ln|x| + \frac{1}{2}u^2 + C$

$= \boxed{\ln|x| + \frac{1}{2}(\ln x)^2 + C}$

l. $\int \sqrt[3]{x^5} + \sqrt[3]{4}\,dx$ $= \int (x^{5/3} + 4^{1/3})dx$ $= \boxed{\frac{3}{8}x^{8/3} + 4^{1/3}x + C}$