Name: ______________________________

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\displaystyle\int_1^9 \frac{x+1}{\sqrt{x}}\,dx = \int_1^9 \left(x^{1/2} + x^{-1/2}\right)dx = \frac{2}{3}x^{3/2} + 2x^{\frac{1}{2}}\Big]_1^9$

$= \left(\frac{2}{3}(9)^{3/2} + 2(9)^{1/2}\right) - \left(\frac{2}{3}(1)^{3/2} + 2(1)^{1/2}\right) = (2\cdot 9 + 2\cdot 3) - \left(\frac{2}{3} + 2\right)$

$= 22 - \frac{2}{3} = 21\frac{1}{3} = \boxed{\frac{64}{3}}$

18
6
24
-2
22

b. $\displaystyle\int_0^{1/2} (2 - \sin(\pi x))\,dx = 2x + \frac{1}{\pi}\cos(\pi x)\Big]_0^{\frac{1}{2}} = \left(2\cdot\frac{1}{2} + \frac{1}{\pi}\cos\left(\frac{\pi}{2}\right)\right) - \left(0 + \frac{1}{\pi}\cos(0)\right)$

$= (1 + 0) - \left(0 + \frac{1}{\pi}\right) = \boxed{1 - \frac{1}{\pi}}$

c. $\displaystyle\int (x+1)(2x+3)\,dx = \int \left(2x^2 + 5x + 3\right)dx = \boxed{\frac{2}{3}x^3 + \frac{5}{2}x^2 + 3x + C}$

expand:
$2x^2 + 2x + 3x + 3$

d. $\int \frac{e^x}{(5+e^x)^4}\,dx$ $= \int u^{-4}\,du = -\frac{1}{3}u^{-3}+C = \boxed{-\frac{1}{3}(5+e^x)^{-3}+C}$

let $u = 5+e^x$

$du = e^x\,dx$

e. $\int \frac{1-\sin(x)}{x+\cos(x)}\,dx$ $= \int \frac{du}{u} = \ln|u|+C = \boxed{\ln|x+\cos x|+C}$

let $u = x+\cos x$

$du = (1-\sin x)\,dx$

f. $\int xe^{2x^2}\,dx$ $= \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C = \boxed{\frac{1}{4}e^{2x^2}+C}$

let $u = 2x^2$

$du = 4x\,dx$

$\frac{1}{4}\,du = x\,dx$

g. $\int x(x+1)^{12}\,dx = \int (u-1)(u)^{12}\,du = \int (u^{13}-u^{12})\,du$

let $u = x+1$

$du = dx$

$x = u-1$

$= \frac{1}{14}u^{14} - \frac{1}{13}u^{13} + C = \boxed{\frac{(x+1)^{14}}{14} - \frac{(x+1)^{13}}{13} + C}$

h. $\int \sec(1-3x)\tan(1-3x)\,dx = \frac{-1}{3}\int \sec u \tan u\,du = \frac{-1}{3}\sec u + C$

let $u = 1-3x$

$du = -3dx$

$-\frac{1}{3}du = dx$

$= \boxed{\frac{-1}{3}\sec(1-3x) + C}$

i. $\int \frac{8}{1+x^2}\,dx = \boxed{8\arctan x + C}$

j. $\int \sqrt{3}\sec^2(x)\,dx$ $= \boxed{\sqrt{3}\tan x + C}$

k. $\int \left(\sqrt[3]{x^4} + \sqrt[3]{5}\right)\,dx$ $= \int \left(x^{\frac{4}{3}} + 5^{1/3}\right)dx = \boxed{\frac{3}{7}x^{\frac{7}{3}} + 5^{1/3}x + C}$

l. $\int \left(\frac{1}{x} + \frac{\ln(x)}{x}\right)\,dx$ $= \ln|x| + \int \frac{\ln x}{x}dx = \ln|x| + \int u\,du = \ln|x| + \frac{1}{2}u^2 + C$

let $u = \ln x$

$du = \frac{1}{x}dx$

$= \boxed{\ln|x| + \frac{1}{2}(\ln x)^2 + C}$

alternative:

$\int \frac{1+\ln(x)}{x}dx = \int u\,du = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}(1+\ln(x))^2 + C}$

pick $u = 1 + \ln(x)$

$du = \frac{1}{x}dx$