Name: ______________________________ **Instructor (circle):** **Maxwell** **Jurkowski** **Sus**

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int_1^2 \frac{2+x^3}{x^2}\,dx = \int_1^2 (2x^{-2}+x)\,dx = -2x^{-1}+\frac{1}{2}x^2\Big]_1^2 = \left(-2\cdot\frac{1}{2}+\frac{1}{2}(2)^2\right)-\left(-2\cdot\frac{1}{1}+\frac{1}{2}(1)^2\right)$

$= -1+2+2-\frac{1}{2} = 4-1.5 = \boxed{2.5}$

b. $\int_0^\pi \left(6x+\sin\left(\frac{x}{2}\right)\right)dx = 3x^2-2\cos\left(\frac{x}{2}\right)\Big]_0^\pi = \left(3\cdot\pi^2-2\cos\left(\frac{\pi}{2}\right)\right)-\left(3\cdot 0^2-2\cos(0)\right)$

$= 3\pi^2-0-0+2 = \boxed{3\pi^2+2}$

c. $\int 10x^2(x-5)\,dx = \int(10x^3-50x^2)\,dx = \boxed{\frac{10}{4}x^4-\frac{50}{3}x^3+C}$

d. $\int e^x \cos(1+e^x)\,dx = \int \cos(u)\,du = \sin(u) + C$

let $u = (1+e^x)$

$du = e^x\,dx$

$$\boxed{= \sin(1+e^x) + C}$$

e. $\int \frac{1}{x^5} + \frac{\sqrt{x}}{5}\,dx = \int \left(x^{-5} + \frac{1}{5}x^{1/2}\right)dx = -\frac{1}{4}x^{-4} + \frac{1}{5}\cdot\frac{2}{3}\cdot x^{3/2} + C$

$$\boxed{= -\frac{1}{4}x^{-4} + \frac{2}{15}x^{3/2} + C}$$

f. $\int \frac{e^{3x}}{\sqrt{5+e^{3x}}}\,dx = \int e^{3x}(5+e^{3x})^{-1/2}\,dx = \frac{1}{3}\int u^{-1/2}\,du = \frac{1}{3}\cdot 2\cdot u^{1/2} + C$

let $u = 5+e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{1}{3}du = e^{3x}\,dx$

$$\boxed{= \frac{2}{3}(5+e^{3x})^{1/2} + C}$$

g. $\int \frac{1}{x} + \sec(x)\tan(x)\,dx$ = $\boxed{\ln|x| + \sec(x) + C}$

h. $\int \left(\frac{1}{\sqrt{1-x^2}} + \frac{1-x^2}{3}\right) dx = \int\left(\frac{1}{\sqrt{1-x^2}} + \frac{1}{3} - \frac{1}{3}x^2\right) dx$

$$\boxed{= \arcsin(x) + \frac{1}{3}x - \frac{1}{9}x^3 + C}$$

i. $\int \frac{3x}{x^2+1}\,dx = \frac{3}{2}\int \frac{du}{u} = \frac{3}{2}\ln|u| + C$

let $u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\boxed{= \frac{3}{2}\ln\left(x^2+1\right) + C}$$

j. $\int x\sqrt{2-x}\,dx$ $= -\int (2-u)u^{1/2}\,du = -\int \left(2u^{1/2} - u^{3/2}\right)du$

let $u = 2-x$

$du = -dx$

$-du = dx$

$x = 2-u$

$= -\left(2\cdot\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right) + C$

$= \boxed{-\frac{4}{3}(2-x)^{3/2} + \frac{2}{5}(2-x)^{5/2} + C}$

k. $\int \tan(x)\sec^2(x)\,dx$ $= \int u\,du = \frac{1}{2}u^2 + C$ $\boxed{= \frac{1}{2}(\tan(x))^2 + C}$

let $u = \tan(x)$

$du = \sec^2(x)\,dx$

l. $\int \frac{x+e^{-x}}{8}\,dx$ $= \frac{1}{8}\int x + e^{-x}\,dx$ $\boxed{= \frac{1}{16}x^2 - \frac{1}{8}e^{-x} + C}$