

Name: _____

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = e^{(3-x^4)}$

$$f'(x) = -4x^3 e^{3-x^4}$$

b. $f(x) = \frac{\sin x}{x^2} = x^{-2} \sin(x)$

$$f'(x) = -2x^{-3} \sin(x) + x^{-2} \cos(x)$$

c. $f(x) = \ln(\sec x + \tan x)$

$$f'(x) = \frac{\sec(x)\tan(x) + \sec^2(x)}{\sec x + \tan(x)}$$

$$d. f(x) = \frac{x^3}{4} + \frac{2}{\sqrt{x}} + \sqrt{50} = \frac{1}{4}x^3 + 2x^{-1/2} + \sqrt{50}$$

$$f'(x) = \frac{3}{4}x^2 + 2\left(-\frac{1}{2}\right)x^{-3/2} + 0$$

$$= \frac{3}{4}x^2 - x^{-3/2}$$

$$e. f(x) = \log_b(x^2 \sin x) \text{ (where } b > 1)$$

$$f'(x) = \frac{2x \sin(x) + x^2 \cos(x)}{(\ln b)(x^2 \sin(x))}$$

$$f. f(x) = (e^x + \cos(2x))^{5/4}$$

$$f'(x) = \frac{5}{4} (e^x + \cos(2x))^{1/4} (e^x - 2 \sin(2x))$$

$$g. y = \pi \left(\frac{x+2}{2} \right)^3 = \pi \left(\frac{x}{2} + 1 \right)^3$$

$$y' = \pi \cdot 3 \left(\frac{x}{2} + 1 \right)^2 \left(\frac{1}{2} \right) = \boxed{\frac{3\pi}{2} \left(\frac{x}{2} + 1 \right)^3}$$

$$h. f(x) = \arctan(\sqrt{x})$$

$$f'(x) = \frac{1}{1 + (\sqrt{x})^2} \cdot \left(\frac{1}{2} x^{-1/2} \right) = \boxed{\frac{1}{2\sqrt{x}(1+x)}}$$

$$i. f(x) = \frac{8+x^2}{x \cos(\pi)}$$

$$f'(x) = \frac{x \cos(\pi) [2x] - (8+x^2) (\cos(\pi))}{[x \cdot \cos(\pi)]^2}$$

$$j. f(x) = x \ln\left(5 + \frac{x}{5}\right) = x \ln\left(5 + \frac{1}{5}x\right)$$

$$f'(x) = 1 \cdot \ln\left(5 + \frac{1}{5}x\right) + x \cdot \left(\frac{\frac{1}{5}}{5 + \frac{1}{5}x}\right)$$

$$= \ln\left(5 + \frac{x}{5}\right) + \frac{x}{25 + x}$$

$$k. f(x) = e^{-x} + e^2 + x^{0.8}$$

$$f'(x) = -e^{-x} + 0.8x^{-0.2}$$

l. Find $\frac{dy}{dx}$ for $x^2 + y^2 = 25 + 2xy^2$. You must solve for $\frac{dy}{dx}$.

$$2x + 2y \frac{dy}{dx} = 0 + 2 \cdot y^2 + 4x \cdot y \frac{dy}{dx}$$

$$(2y - 4xy) \left(\frac{dy}{dx}\right) = 2y^2 - 2x$$

$$\frac{dy}{dx} = \frac{2y^2 - 2x}{2y - 4xy}$$