

Name: _____

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{\cos x}{x^3} = x^{-3} \cos(x)$

$$f'(x) = -3x^{-4} \cos(x) + x^{-3} (-\sin(x))$$

$$= \boxed{-\left(3x^{-4} \cos(x) + x^{-3} \sin(x)\right)}$$

b. $f(x) = e^{(4-x^5)}$

$$f'(x) = -5x^4 e^{4-x^5}$$

• c. $f(x) = (\sin(4x) + e^x)^{6/5}$

$$f'(x) = \frac{6}{5} (\sin(4x) + e^x)^{\frac{1}{5}} (4 \cos(4x) + e^x)$$

d. $f(x) = \ln(\sec x + \tan x)$

$$f'(x) = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$$

e. $f(x) = \frac{x^3}{2} + \frac{7}{\sqrt{x}} + \sqrt{30} = \frac{1}{2}x^3 + 7x^{-1/2} + \sqrt{30}$

$$f'(x) = \frac{3}{2}x^2 - \frac{7}{2}x^{-3/2}$$

f. $f(x) = \log_b(x \cos x)$ (where $b > 1$)
 $= \log_b x + \log_b(\cos x)$

$$f'(x) = \frac{1}{x \ln b} + \frac{-\sin(x)}{(\ln b) \cos(x)}$$

$$g. f(x) = \frac{1+x^4}{x \tan(\pi/3)} = \frac{1}{\tan(\pi/3)} \cdot x^{-1} (1+x^4) = \frac{1}{\tan(\pi/3)} (x^{-1} + x^3)$$

$$f'(x) = \frac{1}{\tan(\pi/3)} (-x^{-2} + 3x^2)$$

$$h. y = \pi \left(\frac{x+8}{5} \right)^2 = \pi \left(\frac{1}{5}x + \frac{8}{5} \right)^2$$

$$\begin{aligned} \frac{dy}{dx} &= 2\pi \left(\frac{1}{5}x + \frac{8}{5} \right) \left(\frac{1}{5} \right) \\ &= \frac{2\pi}{25} (x+8) \end{aligned}$$

$$i. f(x) = \arctan(\sqrt{x})$$

$$\begin{aligned} f'(x) &= \frac{1}{1+(\sqrt{x})^2} \cdot \left(\frac{1}{2} x^{-1/2} \right) \\ &= \frac{1}{2\sqrt{x}(1+x)} \end{aligned}$$

$$j. f(x) = x^2 \ln\left(6 + \frac{x}{6}\right) = x^2 \ln\left(6 + \frac{1}{6}x\right)$$

$$f'(x) = 2x \ln\left(6 + \frac{1}{6}x\right) + x^2 \cdot \left(\frac{\frac{1}{6}}{6 + \frac{1}{6}x}\right)$$

$$= 2x \ln\left(6 + \frac{1}{6}x\right) + \frac{x^2}{36 + x}$$

$$k. f(x) = x^{0.7} + e^2 + e^{-x}$$

$$f'(x) = 0.7x^{-0.3} + 0 - e^{-x}$$

$$= 0.7x^{-0.3} - e^{-x}$$

l. Find $\frac{dy}{dx}$ for $x^2 + y^2 = 25 + 2xy^3$. You must solve for $\frac{dy}{dx}$.

$$2x + 2y \frac{dy}{dx} = 0 + 2 \cdot y^3 + 2x \cdot 3y^2 \frac{dy}{dx}$$

$$(2y - 6xy^2) \frac{dy}{dx} = 2y^3 - 2x$$

$$\frac{dy}{dx} = \frac{2y^3 - 2x}{2y - 6xy^2}$$