

Name: _____

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- A passing score is 10/12.
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{1}{2x} + \sqrt{2x} = \frac{1}{2} x^{-1} + \sqrt{2} x^{\frac{1}{2}}$

$$f'(x) = -\frac{1}{2} x^{-2} + \frac{\sqrt{2}}{2} x^{-\frac{1}{2}}$$

b. $f(x) = a^{\sin(x)}$ where a is a constant, $a > 1$

$$f'(x) = (\ln a) a^{\sin(x)} (\cos(x))$$

c. $f(x) = \sqrt{x + \ln(2x)} = (x + \ln(2x))^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2} (x + \ln(2x))^{-\frac{1}{2}} \left(1 + \frac{2}{2x}\right)$$

d. $f(x) = 1 - x^2 + \sin(1.7x)$

$$f'(x) = -2x + 1.7 \cos(1.7x)$$

e. $y = \sin^{-1}(\sqrt{x})$

$$y' = \frac{1}{\sqrt{1-x}}$$

f. $f(x) = \sec\left(\frac{x}{x+1}\right)$

$$f'(x) = \sec\left(\frac{x}{x+1}\right) \tan\left(\frac{x}{x+1}\right) \left[\frac{(x+1)(1) - x(1)}{(x+1)^2} \right]$$

g. $f(x) = \sqrt{1+x^3} = (1+x^3)^{1/2}$

$$f'(x) = \frac{1}{2}(1+x^3)^{-1/2} (3x^2) = \frac{3x^2}{2\sqrt{1+x^3}}$$

h. $f(x) = \frac{e^x}{x^3} = x^{-3}e^x$

$$f'(x) = -3x^{-4}e^x + x^{-3}e^x$$

OR quotient rule

$$f'(x) = \frac{x^3 e^x - e^x (3x^2)}{x^6} = \frac{e^x (x^3 - 3x^2)}{x^6}$$

i. $f(x) = (\ln(x^2 + e^2))^5$

$$\begin{aligned} f'(x) &= 5 (\ln(x^2 + e^2))^4 \left(\frac{1}{x^2 + e^2} \right) (2x) \\ &= \frac{10x (\ln(x^2 + e^2))^4}{x^2 + e^2} \end{aligned}$$

j. $f(x) = \frac{x \ln(x)}{2}$

$$f'(x) = \frac{1}{2} \left(\ln(x) + \frac{x}{x} \right)$$

k. $f(x) = e^{\pi x + 1} + \sqrt{3} \tan(\pi x)$

$$f'(x) = \pi e^{\pi x + 1} + \sqrt{3} \pi \sec^2(\pi x)$$

l. Find $\frac{dy}{dx}$ for $2x + y = \cos(xy)$. You must solve for $\frac{dy}{dx}$.

$$2 + \frac{dy}{dx} = -\sin(xy) \left(1 \cdot y + x \cdot \frac{dy}{dx} \right)$$

$$\frac{dy}{dx} (1 + x \sin(xy)) = -y \sin(xy) - 2$$

$$\frac{dy}{dx} = \frac{-y \sin(xy) - 2}{1 + x \sin(xy)}$$