Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{2x}{3} + \frac{2}{3x} - \frac{2\pi}{3} = \frac{2}{3}x + \frac{2}{3}x^{-1} - \frac{2\pi}{3}$

$f'(x) = \frac{2}{3} - \frac{2}{3}x^{-2}$

b. $G(\theta) = \theta^2 \tan(\theta)$

$G'(\theta) = 2\theta \tan(\theta) + \theta^2 \cdot \sec^2\theta$

c. $h(x) = \sqrt{x^4 - 16} = (x^4-16)^{\frac{1}{2}}$

$h'(x) = \frac{1}{2}(x^4-16)^{-1/2}(4x^3)$

$= \frac{4x^3}{2\sqrt{x^4-16}} = \frac{2x^3}{\sqrt{x^4-16}}$

d. $y = \cot(x) = \dfrac{\cos(x)}{\sin(x)}$

$y' = -\csc^2(x)$ OR $y' = \dfrac{\sin(x)(-\sin(x)) - \cos(x)(\cos(x))}{\sin^2(x)}$

$= \dfrac{-1}{\sin^2(x)} = -\csc^2(x)$

e. $k(x) = \arcsin(4x)$

$K'(x) = \dfrac{1}{\sqrt{1-(4x)^2}} \cdot 4 = \dfrac{4}{\sqrt{1-16x^2}}$

f. $R(\theta) = \left(2\theta + \cos\left(\dfrac{\theta}{\pi}\right)\right)^5$

$R'(\theta) = 5\left(2\theta + \cos\left(\frac{1}{\pi}\theta\right)\right)^4 \cdot \left[2 + \left(-\sin\left(\frac{1}{\pi}\theta\right)\right)\left(\frac{1}{\pi}\right)\right]$

$= 5\left(2 - \dfrac{\sin\left(\frac{1}{\pi}\theta\right)}{\pi}\right)\left(2\theta + \cos\left(\frac{1}{\pi}\theta\right)\right)^4$

g. $y = (7x-1)^{-2/3}\ln(x)$

$$y' = -\frac{2}{3}(7x-1)^{-5/3}(7)\cdot\ln(x) + (7x-1)^{-2/3}\left(\frac{1}{x}\right)$$

$$= -\frac{14}{3}(7x-1)^{-5/3}\ln(x) + \frac{(7x-1)^{-2/3}}{x}$$

h. $y = \ln(5) + e^{5x} + \sec(2x)$

$$y' = 5e^{5x} + 2\sec(2x)\tan(2x)$$

i. $f(x) = \left(b^2 + \ln(bx^2+1)\right)^{7.8}$ (Assume b is a fixed constant.)

$$f'(x) = 7.8\left(b^2 + \ln(bx^2+1)\right)^{6.8}\cdot\left[\frac{2bx}{bx^2+1}\right]$$

j. $y = \dfrac{5e^x}{x - e^x}$

$$y' = \frac{(x-e^x)(5e^x) - 5e^x(1-e^x)}{(x-e^x)^2} = \frac{5e^x(x-e^x-1+e^x)}{(x-e^x)^2}$$

$$= \frac{5e^x(x-1)}{(x-e^x)^2}$$

k. $f(x) = x\left(\dfrac{2x - x^{-2}}{3x^2}\right) = \frac{1}{3}\cdot x^1 \cdot (2x - x^{-2})(x^{-2}) = \frac{1}{3}(2x^2 - x^{-1})(x^{-2}) = \frac{1}{3}(2 - x^{-3})$

$$f'(x) = \frac{1}{3}(3x^{-4}) = x^{-4}$$

OR product + quotient rule

$$f'(x) = 1\left(\frac{2x - x^{-2}}{3x^2}\right) + x\left(\frac{3x^2(2+2x^{-3}) - (2x-x^{-2})(6x)}{(3x^2)^2}\right)$$

l. Find $\dfrac{dy}{dx}$ for $\sin(y^2) = x + y + \sqrt{2}$.

$$\sin(y^2) = x + y + \sqrt{2}$$

$$\cos(y^2)\cdot 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\left(2y\cos(y^2) - 1\right)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y\cos(y^2) - 1}$$