Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{2x}{5} + \frac{2}{5x} - \frac{2\pi}{5} = \frac{2}{5}x + \frac{2}{5}x^{-1} - \frac{2\pi}{3}$

$f'(x) = \frac{2}{5} - \frac{2}{5}x^{-2}$

b. $h(x) = \sqrt{x^2 - 25} = (x^2-25)^{1/2}$

$h'(x) = \frac{1}{2}(x^2-25)^{-1/2}(2x)$

c. $G(\theta) = \theta^4 \tan(\theta)$

$G'(\theta) = 4\theta^3 \tan(\theta) + \theta^4 \sec^2\theta$

d. $k(x) = \arcsin(3x)$

$$k'(x) = \frac{1}{\sqrt{1-(3x)^2}}(3) = \frac{3}{\sqrt{1-9x^2}}$$

e. $R(\theta) = \left(2\theta + \sin\left(\frac{\theta}{\pi}\right)\right)^6$

$$R'(\theta) = 6\left(2\theta + \sin\left(\tfrac{1}{\pi}\theta\right)\right)^5 \cdot \left[2 + \tfrac{1}{\pi}\cos\left(\tfrac{1}{\pi}\theta\right)\right]$$

f. $y = \cot(x) = \frac{\cos(x)}{\sin(x)}$

$$y' = -\csc^2 x$$

$$y' = \frac{\sin(x)(-\sin(x)) - \cos(x)(\cos(x))}{\sin^2(x)}$$

$$= \frac{-1}{\sin^2(x)} = -\csc^2(x)$$

g. $f(x) = \left(c^2 + \ln(cx^2+1)\right)^{6.5}$ (Assume c is a fixed constant.)

$$f'(x) = 6.5\left(c^2 + \ln(cx^2+1)\right)^{5.5} \cdot \left[\frac{2cx}{cx^2+1}\right]$$

h. $y = (4x-1)^{-1/5}\ln(x)$

$$y' = -\frac{1}{5}(4x-1)^{-6/5}(4)\ln(x) + (4x-1)^{-\frac{1}{5}} \cdot \left(\frac{1}{x}\right)$$

i. $y = \ln(7) + e^{7x} + \sec(5x)$

$$y' = 7e^{7x} + 5\sec(5x)\tan(5x)$$

j. $f(x) = x\left(\frac{3x - x^{-2}}{2x^2}\right) = \frac{3x^2 - x^{-1}}{2x^2} = \frac{3}{2} - \frac{1}{2}x^{-3}$

$$f'(x) = \frac{3}{2}x^{-4}$$

or product + quotient

$$f'(x) = 1\left(\frac{3x - x^{-2}}{2x^2}\right) + x\left(\frac{2x^2(3 + 2x^{-3}) - (3x - x^{-2})(4x)}{(2x^2)^2}\right)$$

k. $y = \frac{8e^x}{x - e^x}$

$$y' = \frac{(x - e^x)(8e^x) - 8e^x(1 - e^x)}{(x - e^x)^2}$$

l. Find $\frac{dy}{dx}$ for $\cos(y^2) = x + y + \sqrt{2}$.

$$-\sin(y^2)\left(2y\frac{dy}{dx}\right) = 1 + \frac{dy}{dx}$$

$$\left(-2y\sin(y^2) - 1\right)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{-2y\sin(y^2) - 1} = \frac{-1}{2y\sin(y^2) + 1}$$