Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{\sqrt{x}}{3} + \frac{5}{\sqrt{x}} - \frac{\sqrt{\pi}}{3} = \frac{1}{3}x^{\frac{1}{2}} + 5x^{-1/2} - \frac{\sqrt{\pi}}{3}$

$$f'(x) = \frac{1}{3}\left(\frac{1}{2}x^{-\frac{1}{2}}\right) + 5\left(\frac{-1}{2}x^{-3/2}\right)$$

$$= \frac{1}{6}x^{-1/2} - \frac{5}{2}x^{-3/2}$$

b. $g(x) = \ln(\sec(x) + \tan(x))$

$$g'(x) = \frac{\sec(x)\tan(x) + \sec^2(x)}{\sec(x) + \tan(x)}$$

c. $h(\theta) = \frac{\sin(\theta)}{\theta^3} = \theta^{-3}\sin(\theta)$

$$h'(\theta) = -3\theta^{-4}\sin(\theta) + \theta^{-3}\cos\theta$$

d. $y = (\cos(4x)+e^x)^3$

$y' = 3\left(\cos(4x)+e^x\right)^2\left((-\sin(4x))(4)+e^x\right)$

e. $k(x) = \arctan(x^2)$

$k'(x) = \dfrac{1}{1+(x^2)^2}(2x) = \dfrac{2x}{1+x^4}$

f. $r(t) = \dfrac{t^3-5t^2+t^{1/3}}{t} = t^2-5t+t^{-2/3}$

$r'(t) = 2t-5-\frac{2}{3}t^{-5/3}$

g. $f(x) = \sqrt{1+x^a}$ where a is a fixed constant

$$f(x) = (1+x^a)^{1/2}$$

$$f'(x) = \frac{1}{2}(1+x^a)^{-1/2}\left(ax^{a-1}\right)$$

h. $y = \ln\left(\frac{x}{1+2x}\right) = \ln(x) + \ln(1+2x)$

$$y' = \frac{1}{x} + \frac{1}{1+2x}(2) = \frac{1}{x} + \frac{2}{1+2x}$$

i. $y = \sin^5(x+e^{-x}) = \left(\sin(x+e^{-x})\right)^5$

$$y' = 5\left(\sin(x+e^{-x})\right)^4\left(\cos(x+e^{-x})\right)(1-e^{-x})$$

j. $f(x) = \frac{1}{6x^2} + xe^x$ $= \frac{1}{6} x^{-2} + xe^x$

$f'(x) = -\frac{2}{6} x^{-3} + 1 \cdot e^x + xe^x$

$= -\frac{1}{3} x^{-3} + e^x(1+x)$

k. $y = \frac{1}{\sin(x)}$ $= \csc(x)$

$y' = -\csc(x)\cot(x)$

l. Find $\frac{dy}{dx}$ for $e^y + x^3 = 10 + xy$

$e^y \frac{dy}{dx} + 3x^2 = 1 \cdot y + x \cdot \frac{dy}{dx}$

$(e^y - x)\frac{dy}{dx} = y - 3x^2$

$\frac{dy}{dx} = \frac{y - 3x^2}{e^y - x}$