Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \sqrt{3}x + \frac{1}{\sqrt{7x}} - \sqrt{\frac{2}{3}}$ $= (\sqrt{3})x + \frac{1}{\sqrt{7}} x^{-1/2} - \sqrt{\frac{2}{3}}$

$f'(x) = \sqrt{3} + \frac{1}{\sqrt{7}}\left(\frac{-1}{2} x^{-3/2}\right) + 0$

b. $g(x) = e^x \cos(x)$

$g'(x) = e^x(-\sin(x)) + e^x \cos(x)$

c. $h(\theta) = \sec(\frac{\theta}{9})$ $= \sec\left(\frac{1}{9}\theta\right)$

$h'(\theta) = \left[\sec\left(\frac{1}{9}\theta\right)\tan\left(\frac{1}{9}\theta\right)\right]\left(\frac{1}{9}\right)$

d. $y = (x + \ln(x^2 - 4))^3$

$$y' = 3\left(x + \ln(x^2-4)\right)^2\left(1 + \frac{2x}{x^2-4}\right)$$

e. $k(x) = \dfrac{1}{x} + x\arcsin(x)$ $= x^{-1} + x \cdot \arcsin(x)$

$$K'(x) = -x^{-2} + 1 \cdot \arcsin(x) + x\left(\frac{1}{\sqrt{1-x^2}}\right)$$

f. $r(x) = \dfrac{\cos(\pi x)}{e^{2x} + 1}$

$$r'(x) = \frac{\left(e^{2x}+1\right)\left(-\pi \sin(\pi x)\right) - \cos(\pi x)\left(2e^{2x}\right)}{\left(e^{2x}+1\right)^2}$$

g. $f(x) = \left(x^2 + \ln(x)\right)^a$ where a is a fixed constant

$$f'(x) = a\left(x^2 + \ln(x)\right)^{a-1}\left(2x + \frac{1}{x}\right)$$

h. $y = \cot(x)$

$$y' = -\csc^2(x)$$

i. $y = \sin^6(x^2)$ $= \left(\sin(x^2)\right)^6$

$$y' = 6\left(\sin(x^2)\right)^5\left(\cos(x^2)\right)(2x)$$

j. $f(x) = \tan\left(\frac{2-x}{3}\right) = \tan\left(\frac{2}{3} - \frac{1}{3}x\right)$

$$f'(x) = \left(\sec^2\left(\frac{2}{3} - \frac{1}{3}x\right)\right)\left(-\frac{1}{3}\right)$$

k. $y = (\pi - 1)x^{\pi}$

$$y' = (\pi - 1)(\pi)x^{\pi - 1}$$

l. Find $\frac{dy}{dx}$ for $\ln(y) + x = 10 + xy^2$

$$\frac{1}{y} \cdot \frac{dy}{dx} + 1 = 0 + 1 \cdot y^2 + 2xy\frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} - 2xy\frac{dy}{dx} = y^2 - 1$$

$$\frac{dy}{dx}\left(\frac{1}{y} - 2xy\right) = y^2 - 1$$

$$\frac{dy}{dx} = \frac{y^2 - 1}{\frac{1}{y} - 2xy}$$

or

$$\frac{dy}{dx} = \frac{y^3 - y}{1 - 2xy^2}$$