Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a "$+C$" where it does not belong and put a "$+C$" in the correct place at least one time.

1. [12 points] Compute the integrals of the following functions.

a. $\displaystyle\int_0^1 5e^x + \sin(x)\,dx = 5e^x - \cos(x)\Big|_0^1 = \left(5e^1 - \cos(1)\right) - \left(5e^0 - \cos(0)\right)$

$= 5e - \cos(1) - 5 + 1 = 5e - \cos(1) - 4$

b. $\displaystyle\int_0^1 2x\sqrt{x^2+5}\,dx = \int_5^6 u^{\frac{1}{2}}\,du = \frac{2}{3}u^{\frac{3}{2}}\Big]_5^6 = \frac{2}{3}\left(6^{3/2} - 5^{3/2}\right)$

Let $u = x^2+5$

$du = 2x\,dx$

If $x=0$, $u=5$

$x=1$, $u=6$

c. $\displaystyle\int (6+\sec^2(\theta))\,d\theta = 6\theta + \tan(\theta) + C$

d. $\int \frac{2-x+x^4}{x^2}\,dx = \int \left(2x^{-2} - x^{-1} + x^{2}\right)dx = 2\cdot\frac{x^{-1}}{-1} - \ln|x| + \frac{1}{3}x^3 + C$

$= -2x^{-1} - \ln|x| + \frac{1}{3}x^3 + C$

e. $\int \frac{1}{1+4x^2}\,dx = \int \frac{1}{1+(2x)^2}\,dx = \frac{1}{2}\arctan(2x) + C$

pick $u=2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$\frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\arctan(u) + C$

$= \frac{1}{2}\arctan(2x) + C$

f. $\int (x + xe^{5x^2})\,dx = \frac{1}{2}x^2 + \frac{1}{10}e^{5x^2} + C$

$= \int x\,dx + \int x e^{5x^2}\,dx = \frac{1}{2}x^2 + \frac{1}{10}\int e^u\,du = \frac{1}{2}x^2 + \frac{1}{10}e^u + C$

let $u = 5x^2$

$du = 10x\,dx$

$\frac{1}{10}du = x\,dx$

$= \frac{1}{2}x^2 + \frac{1}{10}e^{5x^2} + C$

g. $\int \frac{1+\cos(t)}{\sin(t)+t}\,dt$ $= \ln\left|\sin(t)+t\right| + C$

Let $u = \sin(t)+t$

$du = (\cos(t)+1)\,dt$

$= \int \frac{du}{u} = \ln|u| + c = \ln|\sin(t)+t| + C$

h. $\int \frac{x(x^{1.2}+1)}{8}\,dx$ $= \frac{1}{8}\int (x^{2.2}+x)\,dx = \frac{1}{8}\left(\frac{x^{3.2}}{3.2} + \frac{1}{2}x^{2}\right) + C$

i. $\int x(x-5)^9\,dx$ $= \int (u+5)u^9\,du = \int (u^{10}+5u^9)\,du = \frac{1}{11}u^{11} + \frac{5}{10}u^{10} + C$

let $u = x-5$

$du = dx$

$u+5 = x$

$= \frac{1}{11}(x-5)^{11} + \frac{1}{2}(x-5)^{10} + C$

j. $\int \sec\left(\frac{x}{\pi}\right)\tan\left(\frac{x}{\pi}\right)\,dx = \pi\sec\left(\frac{x}{\pi}\right) + C$

(pick $u = \frac{x}{\pi}$)

$du = \frac{1}{\pi}\,dx$

$\pi\,du = dx$

$= \pi\int \sec u \tan u\,du = \pi\sec u + C = \pi\sec\left(\frac{x}{\pi}\right) + C$

k. $\int \frac{\ln(x)}{x}\,dx = \frac{1}{2}\left(\ln(x)\right)^2 + C$

pick $u = \ln(x)$

So $du = \frac{1}{x}dx$

$= \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\left(\ln(x)\right)^2 + C$

l. $\int \left(\frac{5}{x} + \frac{\cos(x)}{5}\right)dx = 5\ln|x| + \frac{1}{5}\sin(x) + C$