Name: ______________________________

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a "$+C$" where it does not belong and put a "$+C$" in the correct place at least one time.

1. [12 points] Compute the integrals of the following functions.

a. $\int_0^1 4e^x + \sin(x)\,dx$ $= 4e^x - \cos(x)\Big]_0^1 = \left(4e^1 - \cos(1)\right) - \left(4e^0 - \cos(0)\right)$

$= 4e - \cos(1) - 4 + 1 = 4e - \cos(1) - 3$

b. $\int_0^1 2x\sqrt{x^2+3}\,dx$ $= \int_3^4 u^{\frac{1}{2}}\,du = \frac{2}{3}u^{\frac{3}{2}}\Big]_3^4 = \frac{2}{3}\left(4^{3/2} - 3^{3/2}\right)$

Let $u = x^2+3$

$du = 2x\,dx$

If $x=0$, $u=3$

$x=1$, $u=4$

c. $\int (5+\sec^2(\theta))\,d\theta$ $= 5\theta + \tan\theta + C$

d. $\int \frac{1}{1+9x^2}\,dx$ $= \int \frac{1}{1+(3x)^2}\,dx = \frac{1}{3}\arctan(3x) + C$

Pick $u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$= \frac{1}{3}\int \frac{du}{1+u^2} = \frac{1}{3}\arctan(u) + C$

$= \frac{1}{3}\arctan(3x) + C$

e. $\int (x + xe^{2x^2})\,dx$ $= \frac{1}{2}x^2 + \frac{1}{4}e^{2x^2} + C$

$= \int x\,dx + \int xe^{2x^2}\,dx = \frac{1}{2}x^2 + \frac{1}{4}\int e^u\,du = \frac{1}{2}x^2 + \frac{1}{4}e^u + C$

$= \frac{1}{2}x^2 + \frac{1}{4}e^{2x^2} + C$

$u = 2x^2$

$du = 4x\,dx$ or $\frac{1}{4}du = x\,dx$

f. $\int \frac{7 - x + x^4}{x^2}\,dx$ $= \int (7x^{-2} - x^{-1} + x^2)\,dx$

$= -7x^{-1} - \ln|x| + \frac{1}{3}x^3 + C$

g. $\int \sec\left(\frac{x}{\pi}\right)\tan\left(\frac{x}{\pi}\right)\,dx \;=\; \pi\sec\left(\frac{x}{\pi}\right)+C$

Pick $u=\frac{x}{\pi}$

$du=\frac{1}{\pi}dx$

$\pi\,du=dx$

$=\pi\int \sec u \tan u\,du = \pi\sec(u)+C = \pi\sec\left(\frac{x}{\pi}\right)+C$

h. $\int \frac{x(x^{1.3}+1)}{6}\,dx = \frac{1}{6}\int\left(x^{2.3}+x\right)dx = \frac{1}{6}\left(\frac{x^{3.3}}{3.3}+\frac{1}{2}x^2\right)+C$

i. $\int \frac{1+\cos(t)}{\sin(t)+t}\,dt = \ln\left|\sin(t)+t\right|+C$

Let $u=\sin(t)+t$

$du=(\cos(t)+1)\,dt$

$=\int\frac{du}{u} = \ln|u|+C = \ln\left|\sin(t)+t\right|+C$

j. $\int \frac{\ln(x)}{x}\,dx = \frac{1}{2}(\ln(x))^2 + C$

pick $u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$

k. $\int x(x-4)^9\,dx = \int (u+4)\cdot u^9\,du = \int \left(u^{10} + 4u^9\right)du = \frac{1}{11}u^{11} + \frac{4}{10}u^{10} + C$

let $u = x-4$

$du = dx$

$u+4 = x$

$= \frac{1}{11}(x-4)^{11} + \frac{2}{5}(x-4)^{10} + C$

l. $\int \left(\frac{5}{x} + \frac{\cos(x)}{5}\right)dx = 5\ln|x| + \frac{1}{5}\sin(x) + C$