Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a "$+C$" where it does not belong and put a "$+C$" in the correct place at least one time.

1. [12 points] Compute the integrals of the following functions.

a. $\int_0^{\pi/2} e^x + \cos(x)\,dx = e^x + \sin(x) \Big|_0^{\pi/2}$

$= \left(e^{\pi/2} + \sin(\tfrac{\pi}{2})\right) - \left(e^0 + \sin(0)\right) = e^{\frac{\pi}{2}} + 1 - 1 - 0 = e^{\frac{\pi}{2}}$

b. $\int_1^2 \frac{\ln(x)}{2x}\,dx = \frac{1}{2}\int_1^2 \frac{\ln(x)}{x}\,dx = \frac{1}{4}(\ln x)^2 \Big|_1^2 = \frac{1}{4}\left((\ln(2))^2 - (\ln(1))^2\right)$

$= \frac{1}{4}(\ln(2))^2$

c. $\int (5^{2/3} + e^{-x} + e^2 x^2)\,dx = 5^{2/3}x - e^{-x} + \frac{e^2}{3}x^3 + C$

d. $\int \frac{3}{\sqrt{1-x^2}}\,dx$ $= 3\arcsin(x) + C$

e. $\int \sec^2(8\theta)\,d\theta$ $= \frac{1}{8}\tan(8\theta) + C$

f. $\int x\sqrt{x+16}\,dx$ $= \int (u-16)\,u^{1/2}\,du = \int \left(u^{3/2} - 16u^{1/2}\right)du$

$u = x+16$

$du = dx$

$u - 16 = x$

$= \frac{2}{5}u^{5/2} - 16\left(\frac{2}{3}u^{3/2}\right) + C$

$= \frac{2}{5}(x+16)^{5/2} - \frac{32}{3}(x+16)^{3/2} + C$

g. $\int 4(\sin(2x))^5 \cos(2x)\, dx$ $= 2\int u^5\, du = \frac{2}{6} u^6 + C$

Let $u = \sin(2x)$

$du = 2\cos(2x)\, dx$

$= \frac{1}{3}(\sin(2x))^6 + C$

h. $\int \frac{4x^3 - 6}{x}\, dx$ $= \int (4x^2 - 6x^{-1})\, dx$

$= \frac{4}{3}x^3 - 6\ln|x| + C$

i. $\int \frac{-t}{t^2+3}\, dt$ $= -\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2} \ln|t^2+3| + C$

Let $u = t^2 + 3$

$du = 2t\, dt$

$\frac{1}{2} du = t\, dt$

j. $\int \sec(x)\tan(x)e^{\sec(x)}\,dx$ $= e^{\sec(x)} + C$

k. $\int x^{-3}(2x+1)\,dx$ $= \int(2x^{-2}+x^{-3})\,dx$

$= -2x^{-1} - \frac{1}{2}x^{-2} + C$

l. $\int \pi^2\,dx$ $= \pi^2 x + C$