

Name: \_\_\_\_\_

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a "+C" where it does not belong and put a "+C" in the correct place at least one time.

1. [12 points] Compute the integrals of the following functions.

$$\begin{aligned} \text{a. } \int_0^\pi e^x + \sin(x) dx &= e^x - \cos(x) \Big|_0^\pi \\ &= (e^\pi - \cos(\pi)) - (e^0 - \cos(0)) \end{aligned}$$

$$\text{b. } \int_0^2 \frac{4t}{10-t^2} dt = 4 \left(-\frac{1}{2}\right) \int_{10}^6 \frac{du}{u} = -2 \cdot \ln|u| \Big|_{10}^6 = -2(\ln(6) - \ln(10))$$

$$\begin{aligned} u &= 10 - t^2 \\ du &= -2t dt \end{aligned}$$

$$-\frac{1}{2} du = t dt$$

$$\begin{aligned} \text{if } t=0, u &= 10 \\ t=2, u &= 6 \end{aligned}$$

$$\text{c. } \int (5^{2/3} + e^{-x} + e^2 x^2) dx = 5^{2/3} x - e^{-x} + \frac{e^2}{3} x^3 + C$$

$$\text{d. } \int \frac{3}{\sqrt{1-x^2}} dx = 3 \arcsin(x) + C$$

$$\text{e. } \int \sec^2(8\theta) d\theta = \frac{1}{8} \tan(8\theta) + C$$

$$\text{f. } \int x\sqrt{x+16} dx = \int (u-16)u^{\frac{1}{2}} du = \int (u^{\frac{3}{2}} - 16u^{\frac{1}{2}}) du$$

$$u = x + 16$$

$$du = dx$$

$$u - 16 = x$$

$$= \frac{2}{5} u^{\frac{5}{2}} - 16 \left( \frac{2}{3} u^{\frac{3}{2}} \right) + C$$

$$= \frac{2}{5} (x+16)^{\frac{5}{2}} - \frac{32}{3} (x+16)^{\frac{3}{2}} + C$$

$$\text{g. } \int 4(\sin(2x))^5 \cos(2x) dx = 2 \int u^5 du = \frac{2}{6} u^6 + C$$

$$\begin{aligned} \text{let } u &= \sin(2x) \\ du &= 2\cos(2x) dx \end{aligned} \quad = \frac{1}{3} (\sin(2x))^6 + C$$

$$\begin{aligned} \text{h. } \int \frac{4x^3 - 6}{x} dx &= \int (4x^2 - 6x^{-1}) dx \\ &= \frac{4}{3} x^3 - 6 \ln|x| + C \end{aligned}$$

$$\begin{aligned} \text{i. } \int \frac{1}{\sqrt{5x}} dx &= \int \frac{1}{\sqrt{5} \cdot \sqrt{x}} dx = \frac{1}{\sqrt{5}} \int x^{-1/2} dx = \frac{1}{\sqrt{5}} \cdot 2 \cdot x^{1/2} + C \\ &= \frac{2}{\sqrt{5}} x^{1/2} + C \end{aligned}$$

$$\text{j. } \int \sec(x) \tan(x) e^{\sec(x)} dx = e^{\sec(x)} + C$$

$$\begin{aligned} \text{k. } \int x^{-3}(2x+1) dx &= \int (2x^{-2} + x^{-3}) dx \\ &= -2x^{-1} - \frac{1}{2}x^{-2} + C \end{aligned}$$

$$\text{l. } \int \frac{e^{\sqrt{x}}}{5\sqrt{x}} dx = \pi^2 x + C$$