Name: Solutions

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle or box your final answer.
- You must use parentheses correctly. A mis-parenthesized answer is incorrect. Do not write $8x \cdot -x^2$ to indicate $8x(-x^2)$, and definitely do not write $8x \cdot -x^2 + 2$ if you mean $8x(-x^2+2)$.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int (-2x^5 + \sin(x))\, dx$

$= -2\frac{x^6}{6} - \cos(x) + C$

b. $\int \cos(6x)\, dx = \int \cos(u) \cdot \frac{du}{6} = \frac{1}{6}\sin(6x) + C$

$u = 6x$

$\frac{du}{6} = dx$

c. $\int_1^2 x e^{x^2}\, dx = \int_1^4 e^u \cdot \frac{du}{2} = \frac{e^u}{2}\Big|_1^4 = \frac{e^4}{2} - \frac{e}{2}$

$u = x^2$

$du = 2x\, dx \Rightarrow x\,dx = \frac{du}{2}$

$x = 1 \Rightarrow u = 1$

$x = 2 \Rightarrow u = 4$

d. $\int \left(\frac{x}{2} + \frac{4}{x} + \frac{6}{5} \right) dx$

$= \frac{1}{2} \int x \, dx + 4 \int \frac{1}{x} \, dx + \int \frac{6}{5} \, dx$

$= \frac{1}{2} \cdot \frac{x^2}{2} + 4 \cdot \ln|x| + \frac{6}{5} x + c$

e. $\int \frac{1 - 2\sin(2x)}{x + \cos(2x)} \, dx = \int \frac{1}{u} \, du = \ln|x + \cos(2x)| + c$

$u = x + \cos(2x)$

$\frac{du}{dx} = 1 - 2\sin(2x)$.

f. $\int \frac{7}{3x(\ln x)^2} \, dx = \frac{7}{3} \int \frac{1}{x(\ln(x))^2} \, dx = \frac{7}{3} \int \frac{1}{u^2} \, du = \frac{7}{3} \int u^{-2} \, du$

$u = \ln(x)$

$du = \frac{1}{x} \, dx$

$= \frac{7}{3} \frac{u^{-1}}{-1} + c = -\frac{7}{3} \frac{1}{u} + c$

$= \frac{-7}{3 \ln(x)} + c$

g. $\int \frac{1}{\sqrt{1-x^2}}\,dx$

$= \arcsin(x) + c$

h. $\int \frac{\arctan(x)}{1+x^2}\,dx$ (recall $\arctan(x) = \tan^{-1}(x)$)

$u = \arctan(x)$

$du = \frac{1}{1+x^2}\,dx$

So $\int \frac{\arctan(x)}{1+x^2}\,dx = \int u\,du = \frac{u^2}{2} + c$

$= \frac{1}{2}(\arctan(x))^2 + c$

i. $\int (e^{-2x} + \sec(x)\tan(x))\,dx$ $= \int e^{-2x}\,dx + \int \sec(x)\tan(x)\,dx$

$u = -2x$

$du = -2\,dx \Rightarrow$

$dx = \frac{du}{-2}$

$= \int \frac{e^u}{-2}\,du + \sec(x) + c$

$= -\frac{1}{2}e^{-2x} + \sec(x) + c$

j. $\int_{-2}^{1} x(3-x)\,dx$ $= \int_{-2}^{1} 3x - x^2\,dx = \left.\frac{3x^2}{2} - \frac{x^3}{3}\right|_{-2}^{1}$

$= \left[\frac{3}{2}(1)^2 - \frac{1}{3}(1)^3\right] - \left[\frac{3}{2}(-2)^2 - \frac{1}{3}(-2)^3\right] = \frac{3}{2} - \frac{1}{3} - \frac{12}{2} - \frac{8}{3}$

$= -\frac{9}{2} - \frac{9}{3} = \frac{-27}{6} - \frac{18}{6} = \frac{-45}{6} = -\frac{9\cdot 5}{3\cdot 2} = -\frac{15}{2}$

k. $\int \frac{x^4}{\sqrt{6-x^5}}\,dx$ $= -\frac{1}{5}\int \frac{1}{\sqrt{u}}\,du = -\frac{1}{5}\int u^{-1/2}\,du$

$u = 6 - x^5$

$du = -5x^4\,dx \Rightarrow$

$-\frac{1}{5}\,du = x^4\,dx$

$= -\frac{1}{5}\left(\frac{u^{1/2}}{1/2}\right) + c$

$= -\frac{2}{5}\sqrt{6-x^5} + c$

l. $\int \frac{x}{x+2}\,dx$ $= \int \frac{u-2}{u}\,du$

$u = x+2$

$du = dx$

$x = u-2$

$= \int 1 - \frac{2}{u}\,du$

$= u - 2\ln|u| + c$

$= x + 2 - 2\ln|x+2| + c$