Name: Solutions

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle or box your final answer.
- You must use parentheses correctly. A mis-parenthesized answer is incorrect. Do not write $8x\cdot -x^2$ to indicate $8x(-x^2)$, and definitely do not write $8x\cdot -x^2+2$ if you mean $8x(-x^2+2)$.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int \left(\frac{-4}{x^5}+\cos(x)\right)dx$ $= -4\int x^{-5}\,dx + \int \cos(x)\,dx$

$= -\frac{4x^{-3}}{-3} + \sin(x) + C$

b. $\int e^{12x}\,dx$ $= \frac{1}{12}\int e^u\,du = \frac{1}{12}e^{12x} + C$

$u = 12$

$du = 12\,dx$

$\frac{du}{12} = dx$

c. $\int_0^{\sqrt{\pi}} x\sin(x^2)\,dx$ $= \int_{u=0}^{u=\pi} \frac{1}{2}\sin(u)\,du = -\frac{1}{2}\cos(u)\Big|_{u=0}^{u=\pi}$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$x = 0 \Rightarrow u = 0$

$x = \sqrt{\pi} \Rightarrow u = \pi$

$= -\frac{1}{2}\cos(\pi) - \left(-\frac{1}{2}\cos(0)\right)$

$= -\frac{1}{2}(-1) + \frac{1}{2}(1)$

$= 1$

d. $\int \frac{1+\sec^2(3x)}{3x+\tan(3x)}\,dx$ $= \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + c$

$u = 3x + \tan(3x)$ $= \frac{1}{3}\ln|3x + \tan(3x)| + c$

$du = (3 + 3\sec^2(3x))\,dx$

$\frac{du}{3} = 1 + \sec^2(x)\,dx$

e. $\int \left(\frac{x^2}{3} + \frac{5}{x} - \frac{1}{2}\right) dx$

$= \frac{1}{3}\frac{x^3}{3} + 5\ln|x| - \frac{1}{2}x + c$

f. $\int \frac{6}{2x(\ln x)^3}\,dx$ $= \int \frac{3}{x(\ln(x))^3}\,dx$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= 3\int \frac{1}{u^3}\,du$

$= 3\int u^{-3}\,du$

$= 3\,\frac{u^{-2}}{-2} + c = \frac{-3}{2u^2} + c$

$= \frac{-3}{2(\ln(x))^2} + c$

g. $\int \frac{1}{1-(4x)^2}\,dx$ $= \frac{1}{4}\int \frac{1}{1-u^2}\,du$

$u = 4x$

$du = 4\,dx$

$\frac{du}{4} = dx$

$= \frac{1}{4}\arctan(4x) + c$

h. $\int \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx$ (recall $\arcsin(x) = \sin^{-1}(x)$)

$u = \arcsin(x)$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$\int \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx = \int u\,du = \frac{u^2}{2} + c$

$= \frac{1}{2}(\arcsin(x))^2 + c$

i. $\int (e^{-5x} + \cos(3x))\,dx$ $= -\frac{1}{5}\int e^u\,du + \frac{1}{3}\int \cos(v)\,dv$

$u = -5x$ $v = 3x$

$\frac{du}{-5} = dx$ $\frac{dv}{3} = dx$

$= -\frac{1}{5}e^{-5x} + \frac{1}{3}\sin(3x) + c$

j. $\int_{-1}^{1} x(5-x)\,dx = \int_{-1}^{1} 5x - x^2\,dx = \frac{5x^2}{2} - \frac{x^3}{3}\Big|_{-1}^{1}$

$= \left(\frac{5}{2}(1)^2 - \frac{1}{3}\right) - \left(\frac{5}{2} + \frac{1}{3}\right)$

$= \frac{5}{2} - \frac{1}{3} - \frac{5}{2} - \frac{1}{3}$

$= \frac{-2}{3}$

k. $\int \frac{x^2}{\sqrt{1-x^3}}\,dx = -\frac{1}{3}\int \frac{1}{\sqrt{u}}\,du$

$u = 1 - x^3$

$du = -3x^2\,dx$

$\frac{du}{-3} = x^2 dx$

$= -\frac{1}{3}\int u^{-1/2}\,du$

$= -\frac{1}{3}\cdot\frac{u^{1/2}}{1/2} = \frac{-2}{3}u^{1/2} + C$

$= -\frac{2}{3}(1-x^3)^{1/2} + C$

l. $\int \frac{t}{t+3}\,dt = \int \frac{u-3}{u}\,du = \int 1 - \frac{3}{u}\,du$

$u = t+3$

$du = dt$

$t = u-3$

$= u - 3\ln|u| + C$

$= t + 3 - 3\ln|t+3| + C$