Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- You must show sufficient work to justify your final expression. A correct answer for a nontrivial computation with no supporting work will be marked as incorrect.
- Your final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.
- **Draw a box around your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(t) = 4t^9 + \frac{5}{t} + \sqrt{\frac{3}{7}}$ $= 4t^9 + 5t^{-1} + \sqrt{3/7}$

$$\boxed{f'(t) = 4(9t^8) + 5(-t^{-2}) + 0}$$

b. $g(x) = \ln(7x^2) + \cot(x)$ $= \ln(7) + \ln(x^2) + \cot(x)$

$= \ln(7) + 2\ln(x) + \cot(x)$

$$\boxed{g'(x) = 0 + \frac{2}{x} + (-(\csc(x))^2)}$$

or, $g'(x) = \frac{1}{7x^2}(14x) - (\csc(x))^2$

c. $y = e^{2x^3-4}\cos(6x-8)$

$$\boxed{y' = e^{2x^3-4}(-\sin(6x-8)6) + \cos(6x-8)e^{2x^3-4}(6x^2)}$$

d. $h(x) = \dfrac{5\csc(3x)}{11e^x + \sqrt{2}}$

$$\boxed{h'(x) = \frac{(11e^x+\sqrt{2})(-5\csc(3x)\cot(3x)(3)) - 5\csc(3x)(11e^x)}{(11e^x+\sqrt{2})^2}}$$

e. $j(\theta) = \ln(\tan(\theta) + \sin(4\theta))$

$$\boxed{j'(\theta) = \left(\frac{1}{\tan\theta + \sin(4\theta)}\right)\left((\sec\theta)^2 + \cos(4\theta)(4)\right)}$$

f. $f(x) = 3^x(Ax+B)^{-1/2}$, where A and B are fixed constants

$$\boxed{f'(x) = 3^x\left(\frac{-1}{2}(Ax+B)^{-3/2}(A)\right) + (Ax+B)^{-1/2}(3^x \ln(3))}$$

g. $y = \pi\sec(x) + \ln(2)$

$$\boxed{y' = \pi \sec(x)\tan(x)}$$

h. $k(t) = \dfrac{t^2 - 5t + 6}{t^{3/2}}$ $= t^{1/2} - 5t^{-1/2} + 6t^{-3/2}$

$$\boxed{K'(t) = \frac{1}{2}t^{-1/2} - 5\left(-\frac{1}{2}t^{-3/2}\right) + 6\left(-\frac{3}{2}t^{-5/2}\right)}$$

i. $f(h) = \dfrac{h + \log_5(h^2)}{8}$ $= \frac{1}{8}\left(h + \log_5(h^2)\right)$

$= \frac{1}{8}\left(h + 2\log_5(h)\right)$

$$\boxed{f'(h) = \frac{1}{8}\left(1 + \frac{2}{h\ln(5)}\right)}$$

or,

$f'(h) =$

$\frac{1}{8}\left(1 + \frac{1}{h^2\ln(5)} \cdot 2h\right)$

j. $y = \sqrt[3]{e^2 + e^{\sin(x)}}$ $= \left(e^2 + e^{\sin(x)}\right)^{1/3}$

$$\boxed{y' = \frac{1}{3}\left(e^2 + e^{\sin(x)}\right)^{-2/3}\left(0 + e^{\sin(x)}(\cos(x))\right)}$$

k. $f(x) = \arctan(6x)$ (this is the same as writing $f(x) = \tan^{-1}(6x)$)

$$\boxed{f'(x) = \frac{1}{1+(6x)^2}(6)}$$

l. Find $\frac{dy}{dx}$ for $y^4 + \cos(x+y^2) = x^3 - 7$. *[You must solve for* $\frac{dy}{dx}$*.]*

$$4y^3\frac{dy}{dx} - \sin(x+y^2)\left(1 + 2y\frac{dy}{dx}\right) = 3x^2 - 0$$

$$4y^3\frac{dy}{dx} - \sin(x+y^2) - 2y\sin(x+y^2)\frac{dy}{dx} = 3x^2$$

$$4y^3\frac{dy}{dx} - 2y\sin(x+y^2)\frac{dy}{dx} = 3x^2 + \sin(x+y^2)$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2 + \sin(x+y^2)}{4y^3 - 2y\sin(x+y^2)}}$$