Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- You must show sufficient work to justify your final expression. A correct answer for a nontrivial computation with no supporting work will be marked as incorrect.
- Your final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.
- **Draw a box around your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(t) = 7t^8 + \frac{9}{t} + \sqrt{\frac{3}{11}}$ $= 7t^8 + 9t^{-1} + \sqrt{3/11}$

$$\boxed{f'(t) = 7(8t^7) + 9(-t^{-2}) + 0}$$

b. $g(x) = \ln(6x^2) + \tan(x)$ $= \ln(6) + \ln(x^2) + \tan(x)$

$= \ln(6) + 2\ln(x) + \tan(x)$

$$\boxed{g'(x) = \frac{2}{x} + (\sec(x))^2}$$

c. $y = e^{3x^2-4}\sin(12x-3)$

$$\boxed{\frac{dy}{dx} = e^{3x^2-4}(\cos(12x-3)\cdot 12) + \sin(12x-3)e^{3x^2-4}(6x)}$$

d. $h(x) = \dfrac{7\sec(3x)}{9e^x + \sqrt{3}}$

$$\boxed{\frac{dh}{dx} = \frac{(9e^x + \sqrt{3})(7\sec(3x)\tan(3x)(3)) - 7\sec(3x)(9e^x)}{(9e^x + \sqrt{3})^2}}$$

e. $j(\theta) = \ln(\cot(\theta) + \cos(5\theta))$

$$\boxed{j'(\theta) = \left(\frac{1}{\cot\theta + \cos(5\theta)}\right)\left(-(\csc(\theta))^2 - \sin(5\theta)\cdot 5\right)}$$

f. $f(x) = 5^x(Ax+B)^{-1/2}$, where A and B are fixed constants

$$\boxed{f'(x) = 5^x\left(-\tfrac{1}{2}(Ax+B)^{-3/2}(A)\right) + (Ax+B)^{-1/2}(5^x \ln 5)}$$

g. $y = \pi \csc(x) + \ln(3)$

$$\boxed{y' = -\pi\left(\csc(x)\cot(x)\right) + 0}$$

h. $k(t) = \frac{t^2 - 4t + 5}{t^{3/2}}$ $= t^{1/2} - 4t^{-1/2} + 5t^{-3/2}$

$$\boxed{k'(t) = \frac{1}{2}t^{-1/2} - 4\left(-\frac{1}{2}t^{-3/2}\right) + 5\left(-\frac{3}{2}t^{-5/2}\right)}$$

i. $f(h) = \frac{h + \log_3(h^2)}{7}$ $= \frac{1}{7}\left(h + 2\log_3(h)\right)$

$$\boxed{f'(h) = \frac{1}{7}\left(1 + \frac{2}{h} \cdot \ln(3)\right)}$$

j. $y = \sqrt[3]{e^2 + e^{\cos(x)}}$ $= \left(e^2 + e^{\cos(x)}\right)^{1/3}$

$$\boxed{y' = \frac{1}{3}\left(e^2 + e^{\cos(x)}\right)^{-2/3}\left(0 + e^{\cos(x)}(-\sin(x))\right)}$$

k. $f(x) = \arctan(5x)$ (this is the same as writing $f(x) = \tan^{-1}(5x)$)

$$\boxed{f'(x) = \frac{1}{1+(5x)^2}(5)}$$

l. Find $\frac{dy}{dx}$ for $y^3 + \cos(x+y^2) = x^4 - 12$. *[You must solve for $\frac{dy}{dx}$.]*

$$3y^2\frac{dy}{dx} - \sin(x+y^2)\left(1 + 2y\frac{dy}{dx}\right) = 4x^3$$

$$3y^2\frac{dy}{dx} - \sin(x+y^2) - 2y\sin(x+y^2)\frac{dy}{dx} = 4x^3$$

$$3y^2\frac{dy}{dx} - 2y\sin(x+y^2)\frac{dy}{dx} = 4x^3 + \sin(x+y^2)$$

$$\boxed{\frac{dy}{dx} = \frac{4x^3 + \sin(x+y^2)}{3y^2 - 2y\sin(x+y^2)}}$$