**Name:** Solutions

Rules:

- One point per problem, 12 points total.
- **No partial credit**.
- Time to complete: 1 hour.
- No aids (book, calculator, etc.) permitted.
- You do **not** need to simplify your expressions.
- Show sufficient work to justify your final expression.
- Final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.

Circle your instructor:

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Compute the derivatives of the following functions. Each problem is worth 1 point for a total of 12 points.

1. $y = e^{x/2}\sin(1-4x)$

• product rule
• chain rule

$$y' = \frac{1}{2}e^{\frac{x}{2}}(\sin(1-4x)) + e^{x/2}(\cos(1-4x))(-4)$$

$$= e^{x/2}\left(\frac{1}{2}\sin(1-4x) - 4\cos(1-4x)\right)$$

2. $f(x) = \dfrac{x-\ln(2)}{5} - \dfrac{1}{6x} = \frac{1}{5}x - \frac{\ln(2)}{5} - \frac{1}{6}x^{-1}$

• algebra first
• Kooky constant

$$f'(x) = \frac{1}{5} + \frac{1}{6}x^{-2}$$

3. $L(t) = \ln(t^2 + \cos^2(t))$

• double chain rule

$$L'(t) = \frac{1}{t^2 + (\cos(t))^2} \cdot \frac{d}{dt}\left[t^2 + (\cos t)^2\right]$$

$$= \left(\frac{1}{t^2 + \cos^2 t}\right)\left(2t + 2\cos(t)(-\sin t)\right)$$

4. $y(x) = \dfrac{\pi \sec(x)}{1+\ln(x)}$

• quotient rule

$$y' = \frac{\pi \sec(x)\tan(x)(1+\ln(x)) - \pi \sec(x)\left(\frac{1}{x}\right)}{(1+\ln(x))^2}$$

• double chain rule

5. $j(\theta) = \tan(\theta - \sqrt[3]{\theta^2+1}) = \tan\left(\theta - (\theta^2+1)^{1/3}\right)$

$$j'(\theta) = \sec^2\left(\theta - (\theta^2+1)^{1/3}\right) \cdot \frac{d}{d\theta}\left(\theta - (\theta^2+1)^{1/3}\right)$$

$$= \sec^2\left(\theta - (\theta^2+1)^{1/3}\right)\left(1 - \frac{1}{3}(\theta^2+1)^{-2/3}(2\theta)\right)$$

6. $y = 4\log_{10}(x^2) + (\sin(x))^{-5} = 8\log_{10} x + (\sin x)^{-5}$

$$y' = 8\left(\frac{1}{\ln(10)\, x}\right) - 5(\sin(x))^{-6}(\cos(x))$$

• logarithm w/ nonstandard base

• negative exponent.

7. $x(\theta) = \arcsin(2\theta)$ (Note: $\arcsin(2\theta)$ is the same as $\sin^{-1}(2\theta)$)

$$x'(\theta) = \frac{1}{\sqrt{1-(2\theta)^2}}(2) = \frac{2}{\sqrt{1-4\theta^2}}$$

- arc trig function

8. $u(x) = (e^2 + e^x)(\sqrt{6} - x^2)$

- product rule
- Kooky constants

$$u'(x) = e^x(\sqrt{6} - x^2) + (e^2 + e^x)(-2x)$$

9. $f(x) = \frac{1}{x^2+1} + \frac{1}{\tan(x)} = (x^2+1)^{-1} + \cot(x)$

- rewrite first
- Kooky trig

$$f'(x) = -(x^2+1)^{-2}(2x) - \csc^2(x)$$

or

$$f'(x) = -(x^2+1)^{-2}(2x) - (\tan(x))^{-2}(\sec^2 x)$$

10. $y = \sqrt{\frac{2^x}{x^3}}$ $= \frac{(2^x)^{1/2}}{(x^3)^{1/2}} = \frac{2^{x/2}}{x^{3/2}} = 2^{x/2}\, x^{-3/2}$

simplify first

$y' = \frac{1}{2}\ln(2)\, 2^{x/2}\, x^{-3/2} + 2^{x/2} \cdot \left(\frac{-3}{2}\right) x^{-5/2}$

OR

as is

$y' = \frac{1}{2}\left(\frac{2^x}{x^3}\right)^{-1/2}\left(\frac{(\ln 2)2^x x^3 - 2^x(3x^2)}{x^6}\right)$

11. $f(x) = x^k + e^{-kx} + 2k$, where $k$ is a fixed constant

$f'(x) = k\, x^{k-1} - k\, e^{-kx}$

12. Find $\frac{dy}{dx}$ for $x^2y^2 + 2x = 2 + \ln(y)$. *[You must solve for $\frac{dy}{dx}$.]*

$2xy^2 + 2x^2y\frac{dy}{dx} + 2 = \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)$

$2xy^2 + 2 = \frac{dy}{dx}\left(\frac{1}{y} - 2x^2y\right)$

$\frac{dy}{dx} = \frac{2xy^2 + 2}{\frac{1}{y} - 2x^2y} = \frac{2xy^3 + 2y}{1 - 2x^2y^2}$