Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a $+C$ where it does not belong, and you must include $+C$ where it is needed.
- You must show sufficient work to justify your final expression. A correct answer for a nontrivial computation with no supporting work will be marked as incorrect.

1. [12 points] Compute the following integrals.

a. $\int (x^5 + e^x - 2x^{-3})\, dx$

$$= \frac{1}{6}x^6 + e^x - 2\left(\frac{-1}{2}\right)x^{-2} + C = \frac{1}{6}x^6 + e^x + x^{-2} + C$$

b. $\int_1^4 \frac{x^2 - 2\sqrt{x}}{x}\, dx = \int_1^4 (x - 2x^{-1/2})\, dx = \frac{1}{2}x^2 - 2(2)x^{\frac{1}{2}}\Big]_1^4$

$$= \left(\frac{1}{2}\cdot 16 - 4\cdot 2\right) - \left(\frac{1}{2} - 4\right) = (8-8) - \left(-\frac{7}{2}\right) = \frac{7}{2}$$

c. $\int e^x \sin(e^x + 1)\, dx = \int \sin(u)\, du = -\cos u + C$

let $u = e^x + 1$

$du = e^x\, dx$

$$= -\cos(e^x + 1) + C$$

d. $\int \pi \left(\frac{x-2}{5}\right) dx$ $= \frac{\pi}{5} \int (x-2)\, dx$

$= \frac{\pi}{5}\left(\frac{1}{2}x^2 - 2x\right) + C$

e. $\int \frac{1+\ln(x)}{3x}\, dx$ $= \frac{1}{3}\int u\, du = \frac{1}{6}u^2 + C$

let $u = 1 + \ln(x)$

$du = \frac{1}{x}\, dx$

$= \frac{1}{6}(1 + \ln(x))^2 + C$

alt. approach: $\frac{1}{3}\int\left(\frac{1}{x} + \frac{\ln x}{x}\right) dx = \frac{1}{3}\left(\ln|x| + \frac{1}{2}(\ln x)^2\right) + C$

f. $\int \left(e^{2x} + \sec^2(3x) + \frac{1}{x}\right) dx$

$= \frac{1}{2}e^{2x} + \frac{1}{3}\tan(3x) + \ln|x| + C$

g. $\int_0^{\pi/2} \frac{5\sin(x)}{\sqrt{1+3\cos(x)}}\,dx$ $= -\frac{5}{3}\int_4^1 u^{-\frac{1}{2}}\,du = -\frac{5}{3}\cdot 2\cdot u^{\frac{1}{2}}\Big]_4^1$

let $u = 1+3\cos(x)$

$du = -3\sin(x)\,dx$

$-\frac{1}{3}du = \sin(x)\,dx$

$= \frac{-10}{3}\left(\sqrt{1}-\sqrt{4}\right) = \frac{-10}{3}(1-2) = \frac{10}{3}$

If $x=0$, $u=4$

If $x=\frac{\pi}{2}$, $u=1$

h. $\int \frac{e^2}{1+x^2}\,dx$ $= e^2\arctan(x) + C$

i. $\int (\cos\theta + \sec\theta\tan\theta + \csc(\pi/4))\,d\theta$

$= \sin(\theta) + \sec(\theta) + \csc(\pi/4)\theta + C$

j. $\int ax^p\,dx$ where a and p are positive constants

$= \frac{a}{p+1} x^{p+1} + C$

k. $\int \frac{5}{3x-1}\,dx = \frac{5}{3} \ln|3x-1| + C$

l. $\int x(x+2)^{10}\,dx = \int (u-2)u^{10}\,du$

$u = x+2$

$du = dx$

$u-2 = x$

$= \int (u^{11} - 2u^{10})\,du$

$= \frac{1}{12} u^{12} - \frac{2}{11} u^{11} = \frac{1}{12}(x+2)^{12} - \frac{2}{11}(x+2)^{11} + C$