Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a $+C$ where it does not belong, and you must include $+C$ where it is needed.
- You must show sufficient work to justify your final expression. A correct answer for a nontrivial computation with no supporting work will be marked as incorrect.

1. [12 points] Compute the following integrals.

a. $\int (x^{-3} - e^x + 2x^5)\,dx$

$= \left(\frac{1}{-2}\right)x^{-2} - e^x + \frac{2}{6}x^6 + C = -\frac{1}{2}x^{-2} - e^x + \frac{1}{3}x^6 + C$

b. $\int \frac{3}{5x-1}\,dx = \frac{3}{5}\int \frac{1}{u}\,du = \frac{3}{5}\ln|u| + C = \frac{3}{5}\ln|5x-1| + C$

let $u = 5x-1$

$du = 5\,dx$

$\frac{1}{5}du = dx$

c. $\int (\sin\theta + \sec\theta\tan\theta + \csc(\pi/4))\,d\theta$

$= -\cos\theta + \sec\theta + \csc\left(\frac{\pi}{4}\right)\theta \quad + C$

d. $\int e^x \cos(e^x+1)\,dx = \int \cos(u)\,du = \sin(u)+C$

let $u = e^x + 1$ $= \sin(e^x+1)+C$

$du = e^x dx$

e. $\int \pi\left(\frac{x-5}{2}\right)dx = \frac{\pi}{2}\int (x-5)\,dx = \frac{\pi}{2}\left(\frac{1}{2}x^2 - 5x\right)+C$

f. $\int \frac{1+\ln(x)}{2x}\,dx = \frac{1}{2}\int u\,du = \frac{1}{4}u^2 + C$

let $u = 1 + \ln(x)$ $= \frac{1}{4}(1+\ln(x))^2 + C$

$du = \frac{1}{x}dx$

alt approach: $\frac{1}{2}\int\left(\frac{1}{x} + \frac{\ln(x)}{x}\right)dx = \frac{1}{2}\left(\ln|x| + \frac{1}{2}(\ln(x))^2\right)+C$

g. $\int \left(\frac{1}{x} + e^{3x} + \sec^2(2x) \right) dx$

$= \ln|x| + \frac{1}{3} e^{3x} + \frac{1}{2} \tan(2x) + C$

h. $\int_0^{\pi/2} \frac{3\sin(x)}{\sqrt{1+5\cos(x)}} dx = -\frac{3}{5} \int_6^1 u^{-1/2} du = -\frac{3}{5} \cdot 2 \cdot u^{\frac{1}{2}} \Big|_6^1$

let $u = 1 + 5\cos(x)$

$du = -5\sin(x)\,dx$

$-\frac{1}{5} du = \sin(x)\,dx$

if $x = 0$, $u = 6$

if $x = \frac{\pi}{2}$, $u = 1$

$= -\frac{6}{5}\left(\sqrt{1} - \sqrt{6}\right)$

$= \frac{6}{5}\left(\sqrt{6} - 1\right)$

i. $\int \frac{e^3}{1+x^2} dx = e^3 \arctan(x) + C$

j. $\displaystyle\int_1^4 \frac{x^2-2\sqrt{x}}{x}\,dx$ $\displaystyle = \int_1^4 \left(x - 2x^{-1/2}\right)dx$

$$= \frac{1}{2}x^2 - 4x^{\frac{1}{2}}\Big]_1^4 = \left(\frac{1}{2}\cdot 16 - 4(2)\right) - \left(\frac{1}{2} - 4\right)$$

$$= (8-8) - \left(\frac{-7}{2}\right) = \frac{7}{2}$$

k. $\displaystyle\int bx^p\,dx$ where b and p are positive constants

$$= \frac{b}{p+1}x^{p+1} + C$$

l. $\displaystyle\int x(x+2)^9\,dx$ $\displaystyle = \int (u-2)u^9\,du = \int \left(u^{10} - 2u^9\right)du$

let $u = x+2$

$du = dx$

$u - 2 = x$

$$= \frac{1}{11}u^{11} - \frac{2}{10}u^{10} + C$$

$$= \frac{1}{11}(x+2)^{11} - \frac{1}{5}(x+2)^{10} + C$$