

Name: Solutions

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- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with $f'(x) =$, $dy/dx =$ or something similar.
- Circle your final answer.

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \pi x^{1/8} + 7e^x + \sqrt{5}$

$$f'(x) = \frac{\pi}{8} x^{-7/8} + 7e^x$$

b. $f(t) = \frac{t^3 - t^{3/2} + 1}{\sqrt{t}}$

$$f(t) = t^{5/2} - t + t^{-1/2}$$

$$f'(t) = \frac{5}{2} t^{3/2} - 1 - \frac{1}{2} t^{-3/2}$$

c. $f(x) = (x^3 - x) \cos(x)$

$$f'(x) = (3x^2 - 1) \cos(x) - (x^3 - x) \sin(x)$$

d. $f(x) = \frac{\sin(x)}{1+e^{-3x}}$

$$f'(x) = \frac{\cos(x)(1+e^{-3x}) + 3\sin(x)e^{-3x}}{(1+e^{-3x})^2}$$

e. $f(x) = \frac{1}{\sin(x)}$

$$f'(x) = \frac{-\cos(x)}{\sin^2(x)} = -\cot(x)\csc(x)$$

either is acceptable

f. $f(t) = t \ln(at)$

$$f'(t) = \ln(at) + t \cdot \frac{1}{at} \cdot a$$

$$= \ln(at) + 1$$

g. $f(x) = \tan(x)x^{\frac{1}{2}}e^{3x}$

$$f'(x) = \sec^2(x)x^{\frac{1}{2}}e^{3x} + \tan(x)\frac{1}{2}x^{-\frac{1}{2}}e^{3x} + 3\tan(x)x^{\frac{1}{2}}e^{3x}$$

h. $f(z) = \arctan(\sqrt{z})$

$$f'(z) = \frac{1}{1+(\sqrt{z})^2} \cdot \frac{1}{2\sqrt{z}}$$

$$= \frac{1}{2(1+z)\sqrt{z}}$$

i. $f(t) = \sec(\ln(1+t^2))$

$$f'(t) = \sec(\ln(1+t^2))\tan(\ln(1+t^2)) \cdot \frac{1}{1+t^2} \cdot 2t$$

j. $f(x) = \sin^5(x^2 + x)$

$$f'(x) = 5 \sin^4(x^2 + x) \cdot \cos(x^2 + x) \cdot (2x + 1)$$

k. $f(x) = \frac{1}{9x} + \left(\pi \frac{x+2}{2}\right)^3$

$$f'(x) = -\frac{1}{9x^2} + 3 \left(\pi \frac{x+2}{2}\right)^2 \cdot \frac{\pi}{2}$$

l. Compute dy/dx if $e^y \sin(x) = 1 - xy$. You must solve for dy/dx .

$$e^y \frac{dy}{dx} \sin(x) + e^y \cos(x) = -y - x \frac{dy}{dx}$$

$$\left[e^y \sin(x) + x \right] \frac{dy}{dx} = -e^y \cos(x) - y$$

$$\frac{dy}{dx} = \frac{-e^y \cos(x) - y}{e^y \sin(x) + x}$$