

Name: Solutions

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- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with $f'(x) =$, $dy/dx =$ or something similar.
- Circle your final answer.

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \sqrt{19}x^{1/3} - 2e^x + \pi$

$$f'(x) = \sqrt{19} \frac{1}{3} x^{-2/3} - 2e^x$$

b. $f(t) = \frac{t^{5/2} + t^2 - t}{\sqrt{t}}$

$$f(t) = t^2 + t^{3/2} - t^{1/2}$$

$$f'(t) = 2t + \frac{3}{2}t^{1/2} - \frac{1}{2}t^{-1/2}$$

c. $f(x) = (x - x^2)\sin(x)$

$$f'(x) = (1 - 2x)\sin(x) + (x - x^2)\cos(x)$$

d. $f(x) = \frac{\cos(x)}{1 + \sin(3x)}$

$$f'(x) = \frac{-\sin(x)(1 + \sin(3x)) - 3\cos(x)\cos(3x)}{(1 + \sin(3x))^2}$$

e. $f(x) = \frac{1}{\sin(x)}$

$$f'(x) = \frac{-\cos(x)}{\sin^2(x)} = -\cot(x)\csc(x)$$

either is acceptable

f. $f(t) = t^2 \ln(at)$

$$f'(t) = 2t \ln(at) + t^2 \frac{1}{at} \cdot a$$

$$= 2t \ln(at) + t$$

g. $f(x) = \sec(x)x^{\frac{1}{3}}e^{4x}$

$$f'(x) = \sec(x)\tan(x)x^{\frac{1}{3}}e^{4x} + \sec(x)\frac{1}{3}x^{-\frac{2}{3}}e^{4x} + \sec(x)x^{\frac{1}{3}}\cdot 4e^{4x}$$

h. $f(z) = \arcsin(\sqrt{z})$

$$f'(z) = \frac{1}{\sqrt{1-(\sqrt{z})^2}} \cdot \frac{1}{2} \frac{1}{\sqrt{z}}$$

$$= \frac{1}{2\sqrt{1-z}\sqrt{z}}$$

i. $f(t) = \tan(\ln(t^3 - 1))$

$$f'(t) = \sec^2(\ln(t^3 - 1)) \cdot \frac{1}{t^3 - 1} \cdot 3t^2$$

j. $f(x) = \cos^4(x^2 - x)$

$$f'(x) = -4 \cos^3(x^2 - x) \cdot \sin(x^2 - x) \cdot (2x - 1)$$

k. $f(x) = \frac{1}{9x^2} + \left(\pi \frac{x-3}{5}\right)^3$

$$f'(x) = \frac{-2}{9x^3} + 3 \left(\pi \frac{x-3}{5}\right)^2 \cdot \frac{\pi}{5}$$

l. Compute dy/dx if $e^y \cos(x) = x^2y - 3$. You must solve for dy/dx .

$$e^y \frac{dy}{dx} \cos(x) - e^y \sin(x) = 2xy + x^2 \frac{dy}{dx}$$

$$\frac{dy}{dx} [e^y \cos(x) - x^2] = 2xy + e^y \sin(x)$$

$$\frac{dy}{dx} = \frac{2xy + e^y \sin(x)}{e^y \cos(x) - x^2}$$