Name: Solutions ______ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with $f'(x) =$, $dy/dx =$ or something similar.
- **Circle your final answer. This is the only expression that will be graded.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \sqrt{5x} - \dfrac{e^x}{2} + \ln 4 = \sqrt{5} \cdot x^{1/2} - \frac{1}{2} e^x + \ln 4$

$$f'(x) = \sqrt{5} \cdot \tfrac{1}{2} \cdot x^{-1/2} - \tfrac{1}{2} e^x = \boxed{\frac{\sqrt{5}}{2\sqrt{x}} - \frac{e^x}{2}}$$

b. $f(t) = \frac{8t + t^{2/3} - 1}{t} = 8 + t^{-1/3} - t^{-1}$

$$\boxed{f'(t) = -\tfrac{1}{3} t^{-4/3} + t^{-2}}$$

c. $h(x) = e^{x/3} \sin(x)$

$$h'(x) = \tfrac{1}{3} e^{\frac{1}{3}x} \cdot \sin x + e^{x/3} (\cos x)$$

$$= \boxed{\tfrac{1}{3} e^{x/3} \sin x + e^{x/3} \cos x}$$

d. $y = (2x^{-1/5} + 6)\ln x$

$$y' = \left(2\cdot\left(-\tfrac{1}{5}\right)x^{-6/5}\right)\cdot \ln x + \left(2x^{-1/5}+6\right)\cdot \frac{1}{x}$$

$$= \boxed{-\frac{2}{5}x^{-6/5}\ln x + 2x^{-6/5} + 6x^{-1}}$$

e. $f(x) = \dfrac{\cos(x)}{\sin(x)}$ $= \cot x$; $\boxed{f'(x) = -\csc^2 x}$

(or use quotient rule...)

$$f'(x) = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

f. $f(x) = x^k + e^{-kx}$, where k is a fixed constant

$$f'(x) = kx^{k-1} + (-k)e^{-kx} = \boxed{k\left(x^{k-1} - e^{-kx}\right)}$$

g. $y = \dfrac{xe^x}{x+1}$

$$y' = \frac{(x+1)[1 \cdot e^x + x \cdot e^x] - (xe^x) \cdot 1}{(x+1)^2} = \frac{e^x[(x+1)^2 - x]}{(x+1)^2} = \boxed{\frac{e^x(x^2+x+1)}{(x+1)^2}}$$

h. $y = \tan\left(x + \sqrt{x}\right)$

$$\boxed{y' = \left[\sec^2\left(x+\sqrt{x}\right)\right] \cdot \left[1 + \tfrac{1}{2}x^{-1/2}\right]}$$

i. $y = 3x + \cos^2(x - 5x^2)$

$$\boxed{y' = 3 - 2 \cdot \cos\left(x-5x^2\right) \cdot \sin\left(x-5x^2\right) \cdot \left(1-10x\right)}$$

j. $f(x) = \ln(x + \sqrt{x^2+1})$

$$f'(x) = \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x\right)$$

$$= \boxed{\left(\frac{1}{x+\sqrt{x^2+1}}\right)\left(1 + \frac{x}{\sqrt{x^2+1}}\right)}$$

k. $g(x) = \arcsin(2x)$

$$g'(x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$$

l. Compute ds/dt if $s^3 e^t + 5 = 2st^2$. You must solve for ds/dt.

$$3s^2 \cdot \frac{ds}{dt} \cdot e^t + s^3 \cdot e^t = 2\frac{ds}{dt}t^2 + 4st$$

$$3s^2e^t\frac{ds}{dt} - 2t^2\frac{ds}{dt} = 4st - s^3e^t$$

$$\frac{ds}{dt} = \frac{4st - s^3e^t}{3s^2e^t - 2t^2}$$