Name: Solutions ________ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with $f'(x) =$, $dy/dx =$ or something similar.
- **Circle your final answer. This is the only expression that will be graded.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \sqrt{6x} - \frac{e^x}{3} + \ln 4 = \sqrt{6} \cdot x^{1/2} - \frac{1}{3} e^x + \ln 4$

$$f'(x) = \sqrt{6} \cdot \frac{1}{2} \cdot x^{-1/2} - \frac{1}{3} e^x = \boxed{\frac{\sqrt{6}}{2\sqrt{x}} - \frac{e^x}{3}}$$

b. $f(t) = \frac{5t - t^{1/3} + 1}{t} = 5 - t^{-2/3} + t^{-1}$

$$f'(t) = -\left(-\frac{2}{3}\right) t^{-5/3} - t^{-2} = \boxed{\frac{2}{3} t^{-5/3} - t^{-2}}$$

c. $h(x) = e^{x/3} \cos(x)$

$$h'(x) = \frac{1}{3} e^{\frac{1}{3}x} \cdot \cos x + e^{x/3} (-\sin x)$$

$$= \boxed{\frac{1}{3} e^{x/3} \cos x - e^{x/3} \sin x}$$

d. $y = (2x^{-2/5} + 6)\ln x$

$$y' = \left(2\cdot\left(-\tfrac{2}{5}\right)x^{-7/5}\right)\cdot \ln x + \left(2x^{-2/5} + 6\right)\cdot \frac{1}{x}$$

$$= \boxed{-\frac{4}{5}x^{-7/5}\ln x + 2x^{-7/5} + 6x^{-1}}$$

e. $f(x) = \dfrac{\cos(x)}{\sin(x)} = \cot x$; $\boxed{f'(x) = -\csc^2 x}$

(or use quotient rule...)

$$f'(x) = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

f. $f(x) = x^k + e^{-kx}$, where k is a fixed constant

$$f'(x) = kx^{k-1} + (-k)e^{-kx} = \boxed{k\left(x^{k-1} - e^{-kx}\right)}$$

g. $y = \dfrac{xe^x}{x+1}$

$$y' = \frac{(x+1)[1 \cdot e^x + x \cdot e^x] - (xe^x) \cdot 1}{(x+1)^2} = \frac{e^x[(x+1)^2 - x]}{(x+1)^2} = \boxed{\frac{e^x(x^2+x+1)}{(x+1)^2}}$$

h. $y = \tan\left(x + \sqrt{x}\right)$

$$\boxed{y' = \left[\sec^2(x+\sqrt{x})\right] \cdot \left[1 + \tfrac{1}{2}x^{-1/2}\right]}$$

i. $y = 3x + \sin^2(x - 5x^2)$

$$\boxed{y' = 3 + 2 \cdot \sin(x-5x^2) \cdot \cos(x-5x^2) \cdot (1-10x)}$$

j. $f(x) = \ln(x + \sqrt{x^2+1})$

$$f'(x) = \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x\right)$$

$$= \boxed{\left(\frac{1}{x+\sqrt{x^2+1}}\right)\left(1 + \frac{x}{\sqrt{x^2+1}}\right)}$$

k. $g(x) = \arccos(2x)$

$$g'(x) = \frac{-1}{\sqrt{1-(2x)^2}} \cdot 2 = \boxed{\frac{-2}{\sqrt{1-4x^2}}}$$

l. Compute ds/dt if $s^2 e^t + 5 = 2st^3$. You must solve for ds/dt.

$$2s\frac{ds}{dt} \cdot e^t + s^2 e^t = 2\frac{ds}{dt}t^3 + 6st^2$$

$$2se^t \frac{ds}{dt} - 2t^3 \frac{ds}{dt} = 6st^2 - s^2 e^t$$

$$\boxed{\frac{ds}{dt} = \frac{6st^2 - s^2e^t}{2se^t - 2t^3}}$$