Name: Solutions ______ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have **60** minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with $f'(x) =$, $dy/dx =$ or something similar.
- Circle your final answer.

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \dfrac{x-\sqrt{3}}{5} - 3x^4 - \sqrt[3]{x}$

$f(x) = \frac{1}{5}x - \frac{\sqrt{3}}{5} - 3x^4 - x^{1/3}$

$f'(x) = \frac{1}{5} - 12x^3 - \frac{1}{3}x^{-2/3}$

b. $y = x^2 \sec(x)$

$y' = 2x\sec(x) + x^2 \sec(x)\tan(x)$

c. $y = \dfrac{\tan(x)}{1+\ln(x)}$

$$y' = \frac{\sec^2(x)[1+\ln(x)] - \tan(x)\frac{1}{x}}{(1+\ln(x))^2}$$

d. $y = e^{ax^2}\cos(bx)$ where a and b are fixed constants.

$$y' = 2ax e^{ax^2}\cos(bx) - be^{ax^2}\sin(bx)$$

e. $f(x) = \arctan(\sin(5x))$

$$f'(x) = \frac{1}{1+\sin^2(5x)} \cdot \left[\cos(5x)\cdot 5\right]$$

f. $g(x) = \sqrt{\sin^2(3x)+1}$

$$g'(x) = \frac{1}{2}\,\frac{1}{\sqrt{\sin^2(3x)+1}} \cdot \left[2\sin(3x)\cdot\cos(3x)\cdot 3\right]$$

g. $y = \tan(xe^x)$

$$y' = \sec^2(xe^x) \cdot \left[e^x + xe^x \right]$$

$$= \sec^2(xe^x) \cdot (1+x)e^x$$

h. $f(x) = \sqrt{x}\,\ln(x)\arcsin(x)$

$$f'(x) = \frac{1}{2}x^{-1/2}\ln(x)\arcsin(x) + \frac{\sqrt{x}}{x}\arcsin(x) + \sqrt{x}\ln(x)\frac{1}{\sqrt{1-x^2}}$$

$$= \frac{1}{2\sqrt{x}}\ln(x)\arcsin(x) + \frac{1}{\sqrt{x}}\arcsin(x) + \ln(x)\sqrt{\frac{x}{1-x^2}}$$

$$= \frac{\arcsin(x)}{\sqrt{x}}\left[\ln(\sqrt{x}) + 1\right] + \ln(x)\sqrt{\frac{x}{1-x^2}}$$

i. $y = \cos\left(\dfrac{x}{x-1}\right)$

$$y' = -\sin\left(\frac{x}{x-1}\right) \cdot \left[\frac{1\cdot(x-1) - x(1)}{(x-1)^2}\right]$$

$$= \sin\left(\frac{x}{x-1}\right)\frac{1}{(x-1)^2}$$

j. $h(x) = \ln\left(\pi x^2 - (4x)^9\right)$

$$h'(x) = \frac{1}{\pi x^2 - (4x)^9} \cdot \left[2\pi x - 9(4x)^8 \cdot 4\right]$$

$$= \frac{2\pi x - 36(4x)^8}{\pi x^2 - (4x)^9}$$

k. $g(x) = \dfrac{e^3}{1-x^2}$

$$g'(x) = \frac{-e^3(-2x)}{(1-x^2)^2} = \frac{2e^3 x}{(1-x^2)^2}$$

l. Compute dy/dx if $\quad 2x^2y^2 - x^3 + y^4 = 0$. You must solve for dy/dx.

$$2xy^2 + 2x^2 y y' - 3x^2 + 4y^3 y' = 0$$

$$\left[2x^2y + 4y^3\right] y' = 3x^2 - 2xy^2$$

$$y' = \frac{3x^2 - 2xy^2}{2x^2y + 4y^3}$$