Name: Solutions ________ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have **60** minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with $f'(x) =$, $dy/dx =$ or something similar.
- Circle your final answer.

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \sqrt[5]{x} + 4x^3 + \dfrac{x-\sqrt{2}}{9}$

$$f(x) = x^{1/5} + 4x^3 + \frac{1}{9}x - \frac{\sqrt{2}}{9}$$

$$f'(x) = \frac{1}{5}x^{-4/5} + 12x^2 + \frac{1}{9}$$

b. $y = x^3 \tan(x)$

$$y' = 3x^2\tan(x) + x^3\sec^2(x)$$

c. $y = \dfrac{\sec(x)}{1+\ln(x)}$

$$y' = \frac{\sec(x)\tan(x)(1+\ln(x)) - \sec(x)\left(\frac{1}{x}\right)}{(1+\ln(x))^2}$$

d. $y = \sin(ax)e^{bx^2}$ where a and b are fixed constants.

$$y' = a\cos(ax)e^{bx^2} + 2bx\sin(ax)e^{bx^2}$$

e. $f(x) = \arcsin(\cos(7x))$

$$f'(x) = \frac{1}{\sqrt{1-\cos^2(7x)}} \cdot \left[-\sin(7x)\cdot 7\right]$$

$$= \frac{-7\sin(7x)}{\sqrt{1-\cos^2(7x)}}$$

f. $g(x) = \sqrt{2+\sin^2(6x)}$

$$g'(x) = \frac{1}{2}\frac{1}{\sqrt{2+\sin^2(6x)}} \cdot \left(2\sin(6x)\cdot\cos(6x)\cdot 6\right)$$

$$= \frac{6\sin(6x)\cos(6x)}{\sqrt{2+\sin^2(6x)}}$$

g. $y = \tan\left(x^3 e^x\right)$

$$y' = \sec^2(x^3e^x) \cdot \left[3x^2e^x + x^3e^x \right]$$

$$= \sec^2(x^3e^x) \left[3x^2 + x^3 \right] e^x$$

h. $f(x) = \sqrt{x}\ \ln(x)\, \arctan(x)$

$$f'(x) = \frac{1}{2\sqrt{x}} \cdot \ln(x) \arctan(x) + \frac{\sqrt{x}}{x} \arctan(x) + \frac{\sqrt{x}\ \ln(x)}{1+x^2}$$

$$= \frac{1}{\sqrt{x}} \left[\ln(\sqrt{x}) + 1 \right] \arctan(x) + \frac{\sqrt{x}\ \ln(x)}{1+x^2}$$

i. $y = \sin\left(\dfrac{x}{x-3}\right)$

$$y' = \cos\left(\frac{x}{x-3}\right) \cdot \frac{x-3 - x(\cdot 1)}{(x-3)^2}$$

$$= \cos\left(\frac{x}{x-3}\right) \cdot \frac{-3}{(x-3)^2}$$

$$= -\frac{3}{(x-3)^2} \cdot \cos\left(\frac{x}{x-3}\right)$$

j. $h(x) = \ln\left(\pi x^3 + (5x)^7\right)$

$$h'(x) = \frac{1}{\pi x^3 + (5x)^7} \cdot \left[3\pi x^2 + 7(5x)^6 \cdot 5\right]$$

$$= \frac{3\pi x^2 + 35(5x)^6}{\pi x^3 + (5x)^7}$$

k. $g(x) = \dfrac{e^5}{3 - x^2}$

$$g'(x) = \frac{-e^5(-2x)}{(3-x^2)^2}$$

$$= \frac{2e^5 x}{(3-x^2)^2}$$

l. Compute dy/dx if $\quad -2x^3 + x^2y^2 + y^5 = 0$. You must solve for dy/dx.

$$-6x^2 + 2xy^2 + x^2 2yy' + 5y^4 y' = 0$$

$$y' = \frac{6x^2 - 2xy^2}{2x^2y + 5y^4}$$