Name: Solutions ______ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int x^{\frac{2}{5}} + \frac{1}{x} + \sqrt{2}\, dx$

$$\frac{5}{7}x^{7/5} + \ln(|x|) + \sqrt{2}\,x + C$$

b. $\int_0^2 e^x + \cos x\, dx$

$$e^x + \sin(x)\Big|_0^2 = e^2 + \sin(2) - e^0 - \sin(0)$$

$$= e^2 + \sin(2) - 1$$

c. $\int \sin(3\pi x)\, dx$

$$\frac{-1}{3\pi}\cos(3\pi x) + C$$

d. $\displaystyle\int \frac{7}{1+x^2}\,dx$

$7\arctan(x) + C$

e. $\displaystyle\int \frac{7x}{1+x^2}\,dx$

$u = 1+x^2$

$du = 2x\,dx$

$$\int \frac{7x}{1+x^2}\,dx = \frac{7}{2}\int \frac{1}{u}\,du = \frac{7}{2}\ln(|u|) + C$$

$$= \frac{7}{2}\ln(1+x^2) + C$$

f. $\displaystyle\int \frac{1+x^2}{7x}\,dx$

$$\int \frac{1+x^2}{7x}\,dx = \int \frac{1}{7x}\,dx + \int \frac{1}{7}x\,dx$$

$$= \frac{1}{7}\ln(|x|) + \frac{1}{14}x^2 + C$$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

g. $\int x + \frac{\ln(x)}{x}\,dx$

$= \frac{x^2}{2} + \int \frac{\ln(x)}{x}\,dx$

$= \frac{x^2}{2} + \int u\,du = \frac{x^2}{2} + \frac{u^2}{2} = \frac{1}{2}\left[x^2 + (\ln(x))^2\right] + C$

h. $\int (1+\tan(x))^2 \sec^2(x)\,dx$

$u = 1 + \tan(x)$

$du = \sec^2(x)$

$\int u^2\,du = \frac{u^3}{3} = \frac{1}{3}(1+\tan(x))^3 + C$

i. $\int x^{\frac{1}{3}}(x+1)\,dx$

$\int x^{1/3}(x+1)\,dx = \int x^{4/3} + x^{1/3}\,dx$

$= \frac{3}{7}x^{7/3} + \frac{3}{4}x^{4/3} + C$

j. $\int x\sqrt{x-3}\,dx$ $= \int (u+3)\sqrt{u}\, du$

$u = x-3$

$du = dx$

$= \int u^{3/2} + 3u^{1/2}\, du$

$= \frac{2}{5}u^{5/2} + 3\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(x-3)^{5/2} + 2(x-3)^{3/2} + C$

k. $\int x^2\sin(x^3)\,dx$ $= \int \frac{1}{3}\sin(u)\,du = -\frac{1}{3}\cos(u) + C$

$u = x^3$

$= -\frac{1}{3}\cos(x^3) + C$

$du = 3x^2dx$

l. $\int \frac{1}{(2x-3)^4}\,dx$ $= \int u^{-4}\,\frac{1}{2}\,du = \frac{1}{2}\frac{1}{(-3)}u^{-3} + C$

$u = 2x-3$

$= -\frac{1}{6}(2x-3)^{-3} + C$

$du = 2dx$