Name: Solutions ______ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int x^{\frac{3}{7}} - \frac{1}{x} + e^2 \, dx$

$$\frac{7}{10} x^{10/7} - \ln(|x|) + xe^2 + C$$

b. $\int_0^2 \sin x + e^x \, dx$

$$-\cos x + e^x \Big|_0^2 = \left(-\cos(2) + e^2\right) - \left(-\cos(0) + e^0\right)$$

$$= -\cos(2) + e^2 + \cos(0) - e^0$$

$$= -\cos(2) + e^2$$

c. $\int \cos(4\pi x) \, dx$

$$\frac{1}{4\pi} \sin(4\pi x)$$

d. $\int \frac{3}{\sqrt{1-x^2}}\,dx$

$3 \arcsin(x) + C$

e. $\int \frac{3x}{1-x^2}\,dx = -\frac{3}{2}\int \frac{1}{u}\,du = -\frac{3}{2}\ln(|u|)$

$= -\frac{3}{2}\ln(|1-x^2|) + C$

$u = 1-x^2$

$du = -2x\,dx$

f. $\int \frac{1-x^2}{3x}\,dx = \int \frac{1}{3x} - \int \frac{1}{3}x\,dx$

$= \frac{1}{3}\ln(|x|) - \frac{1}{6}x^2 + C$

g. $\int e^x + \frac{\ln(x)}{x}\, dx$

$= e^x + \int \frac{\ln(x)}{x}\, dx$

$u = \ln(x)$

$du = \frac{1}{x}$

$\int \frac{\ln(x)}{x}\, dx = \int u\, du = \frac{u^2}{2}$

$= \frac{(\ln(x))^2}{2}$

$= e^x + \frac{(\ln(x))^2}{2} + C$

h. $\int (1+\sec(x))^2 \sec(x)\tan(x)\, dx = \int u^2\, du = \frac{u^3}{3} + C$

$u = 1 + \sec(x)$

$du = \sec(x)\tan(x)\,dx$

$= \frac{1}{3}(1+\sec(x))^3 + C$

i. $\int x^{\frac{2}{3}}(x-1)\, dx = \int x^{5/3} - x^{2/3}\, dx$

$= \frac{3}{8}x^{8/3} - \frac{3}{5}x^{5/3} + C$

j. $\int x\sqrt{x-5}\,dx$ $= \int (u+5)\sqrt{u}\,du$

$u = x-5$

$du = dx$

$= \int u^{3/2} + 5u^{1/2}\,du$

$= \frac{2}{5}u^{5/2} + 5\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(x-5)^{5/2} + \frac{10}{3}(x-5)^{3/2} + C$

k. $\int x^2 e^{x^3}\,dx$ $= \int \frac{1}{3}e^u\,du = \frac{1}{3}e^u = \frac{1}{3}e^{x^3}$

$u = x^3$

$du = 3x^2\,dx$

l. $\int \frac{1}{(3x-2)^3}\,dx$ $= \frac{1}{3}\int \frac{1}{u^3}\,du = \frac{1}{3}\left(\frac{-1}{2}\right)u^{-2} + C$

$u = 3x-2$

$du = 3\,dx$

$= -\frac{1}{6}(3x-2)^{-2} + C$