Name: ______________________________ ________ / **12**

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int_1^4 \left(\frac{1}{x} - \sqrt{x}\right)\,dx$

$$\ln(|x|) - \frac{2}{3}x^{3/2}\Big|_1^4 = \left(\ln 4 - \frac{16}{3}\right) - \left(\ln(1) - \frac{2}{3}\right)$$

$$= \ln(4) - \frac{14}{3}$$

b. $\int \left(7^{\frac{1}{3}} + e^{5x} - \pi x^2\right)\,dx$

$$7^{1/3}x + \frac{1}{5}e^{5x} - \pi\frac{x^3}{3} + C$$

c. $\int \frac{1}{x\ln(x)}\,dx$

$$\int \frac{1}{u}\,du = \ln(|u|) + C$$

$$u = \ln(x)$$

$$du = \frac{1}{x}\,dx$$

$$= \ln(|\ln(x)|) + C$$

d. $\int (x-2)(x-3)\,dx$

$$= \int x^2 - 5x + 6\,dx = \frac{x^3}{3} - \frac{5}{2}x^2 + 6x + C$$

e. $\int \sec^2(x)\,e^{\tan(x)}\,dx$ $= \int e^u\,du = e^u + C$

$u = \tan(x)$ $= e^{\tan(x)} + C$

$du = \sec^2(x)\,dx$

f. $\int \left(\frac{8x}{1-x^2} + \cos(x)\right)\,dx$

$$\int \frac{8x}{1-x^2}\,dx = -4\int \frac{1}{u}\,du = -4\ln(|u|) = -4\ln(|1-x^2|)$$

$u = 1-x^2$

$\int \cos(x)\,dx = \sin(x)$

$du = -2x\,dx$

$$\boxed{-4\ln(|1-x^2|) + \sin(x) + C}$$

g. $\int x\sqrt{x-9}\,dx$ $= \int (u+9)\sqrt{u}\,du$

$u = x-9$

$du = dx$

$= \int u^{3/2} + 9u^{1/2}\,du$

$= \frac{2}{5}u^{5/2} + 9\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(x-9)^{5/2} + 6(x-9)^{3/2} + C$

h. $\int \cos(x)\,(\sin(x)-3)^5\,dx$ $= \int u^5\,du = \frac{u^6}{6} + C$

$u = (\sin(x) - 3)$

$du = \cos(x)\,dx$

$= \frac{(\sin(x)-3)^6}{6} + C$

i. $\int \sec^2\left(\frac{\pi}{2}t\right)\,dt$ $\frac{2}{\pi}\int \sec^2(u)\,du = \frac{2}{\pi}\tan(u) + C$

$u = \frac{\pi}{2}t$

$= \frac{2}{\pi}\tan\left(\frac{\pi}{2}t\right) + C$

$du = \frac{\pi}{2}dt$

j. $\displaystyle\int \frac{6}{\sqrt{1-s^2}}\,ds$

$6 \arcsin(s) + C$

k. $\displaystyle\int e^{-8t+5}\,dt$

$-\frac{1}{8} e^{-8t+5} + C$

l. $\displaystyle\int \frac{2x^3-5}{x}\,dx = \int 2x^2 - \frac{5}{x}\,dx = \frac{2x^3}{3} - 5\ln(|x|) + C$