

Name: SOLUTIONS

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- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with**  $f'(x) =$ ,  $dy/dx =$ , or similar.
- Circle your final answer.

Compute the derivatives of the following functions.

1.  $f(x) = \frac{x - \ln 2}{5} - \sqrt[3]{x}$

$$f'(x) = \frac{1}{5} - \frac{1}{3}x^{-2/3}$$

2.  $g(x) = \frac{1}{\sin(x)} = \csc x$

$$g'(x) = -\csc x \cot x$$

3.  $f(t) = \frac{1 - 4t^{1/2} + t^3}{t} = t^{-1} - 4t^{-1/2} + t^2$

$$f'(t) = -t^{-2} + 2t^{-3/2} + 2t$$

4.  $h(x) = e^{-x/4} \cos(x)$

$$h'(x) = -\frac{1}{4} e^{-x/4} \cos x + e^{-x/4} (-\sin x)$$

5.  $y = \arcsin(2x + \sqrt{6})$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - (2x + \sqrt{6})^2}} \quad (2)$$

6.  $f(x) = x^k + e^{-kx}$ , where  $k$  is a fixed constant

$$f'(x) = kx^{k-1} - ke^{-kx}$$

$$7. y = \frac{\tan(x)}{1 + \ln(x)}$$

$$y' = \frac{\sec^2(x)(1 + \ln x) - \tan(x)\left(\frac{1}{x}\right)}{(1 + \ln(x))^2}$$

$$8. h(x) = \frac{\pi}{x^2} + \left(\frac{x-1}{4}\right)^3 = \pi x^{-2} + \left(\frac{1}{4}(x-1)\right)^3$$

$$h'(x) = -2\pi x^{-3} + 3\left(\frac{1}{4}(x-1)\right)^2 \cdot \frac{1}{4}$$

$$9. y = \sin^2(x - \sqrt{x^2 + 1})$$

$$y' = 2 \sin(x - \sqrt{x^2 + 1}) \cdot \cos(x - \sqrt{x^2 + 1}) \left(1 - \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot 2x\right)$$

10.  $y = e^x \ln(x) \sec(x)$

$$y' = e^x \ln(x) \sec(x) + e^x \cdot \frac{1}{x} \cdot \sec(x) + e^x \ln(x) \cdot \sec(x) \tan(x)$$

11.  $g(x) = \frac{\cos(2x)}{x^3 + x}$

$$g'(x) = \frac{-2\sin(2x) \cdot (x^3 + x) - \cos(2x) (3x^2 + 1)}{(x^3 + x)^2}$$

12. Compute  $dy/dt$  if  $e^y + t^3 = y \cos(y)$ . You must solve for  $dy/dt$ .

$$e^y y' + 3t^2 = y' \cos(y) + y (-\sin(y)) \cdot y'$$

$$y' (e^y - \cos(y) + y \sin(y)) = -3t^2$$

$$\frac{dy}{dt} = \frac{-3t^2}{e^y - \cos(y) + y \sin(y)}$$