

Name: Solutions

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- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with $f'(x) =$, $dy/dx =$ or something similar.
- Circle your final answer.

1. [12 points] Compute the derivatives of the following functions.

a. $f(t) = 2t^{2/3} + \frac{3}{t^{2/3}} + \sqrt{\frac{2}{3}}$

$$f'(t) = \frac{4}{3}t^{-1/3} + 3\left(-\frac{2}{3}\right)t^{-5/3}$$

$$= \frac{4}{3}t^{-1/3} - 2t^{-5/3}$$

b. $r(x) = \sec(x^2 + 1)$

$$\frac{dr}{dx} = \sec(x^2 + 1) \tan(x^2 + 1) (2x)$$

c. $g(x) = (e^{3x} + e) \tan(x)$

$$g'(x) = (3e^{3x}) \tan(x) + (e^{3x} + e) \sec^2(x)$$

d. $h(x) = \ln(B\cos(x^3) - A)$, where A and B are fixed constants

$$h'(x) = \frac{1}{B\cos(x^3) - A} (-B\sin(x^3))(3x^2)$$

$$= \frac{-3x^2 B \sin(x^3)}{B\cos(x^3) - A}$$

e. $f(x) = \frac{1}{\sin(7x)} = \csc(7x)$

$$\frac{df}{dx} = -\csc(7x)\cot(7x)(7)$$

f. $g(t) = (\sqrt{t^2+1})\ln(t)$

$$g'(t) = \frac{1}{2}(t^2+1)^{-\frac{1}{2}}(2t)\ln t + \sqrt{t^2+1}\left(\frac{1}{t}\right)$$

$$= \frac{t \ln t}{\sqrt{t^2+1}} + \frac{\sqrt{t^2+1}}{t}$$

g. $f(x) = (x^3 + 3)e^x \cos(x)$

$$f'(x) = (3x^2)e^x \cos x + (x^3 + 3)(e^x \cos x - e^x \sin x)$$

h. $g(z) = \sin(\pi - z^3)$

$$\frac{dg}{dz} = \cos(\pi - z^3) (-3z^2)$$

i. $s(t) = \frac{\cos(2t)}{t^2 + 2}$

$$s'(t) = \frac{-2\sin(2t)(t^2 + 2) - \cos(2t)(2t)}{(t^2 + 2)^2}$$

j. $f(x) = \frac{2x+5}{2\ln x + \ln 5}$

$$\frac{df}{dx} = \frac{2(2\ln x + \ln 5) - (2x+5)\left(\frac{2}{x}\right)}{(2\ln x + \ln 5)^2}$$

k. $g(x) = \arctan(e^x)$

$$g'(x) = \frac{1}{1+(e^x)^2} e^x = \frac{e^x}{1+e^{2x}}$$

l. Compute $\frac{dy}{dx}$ if $e^{x+y} = xy + 3\cos y$. You must solve for $\frac{dy}{dx}$.

$$e^{x+y} \left(1 + \frac{dy}{dx}\right) = y + x \frac{dy}{dx} - 3\sin y \frac{dy}{dx}$$

$$\left(e^{x+y} - x + 3\sin y\right) \frac{dy}{dx} = y - e^{x+y}$$

$$\frac{dy}{dx} = \frac{y - e^{x+y}}{e^{x+y} - x + 3\sin y}$$