Name: Solutions ______ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with $f'(x) =$, $dy/dx =$ or something similar.
- Circle your final answer.

1. [12 points] Compute the derivatives of the following functions.

a. $q(t) = \left(\sqrt{1+t^4}\right)\ln(t)$

$$q'(t) = \frac{1}{2}(1+t^4)^{-1/2}(4t^3)\ln t + \sqrt{1+t^4}\left(\frac{1}{t}\right)$$

$$= \frac{2t^3 \ln t}{\sqrt{1+t^4}} + \frac{\sqrt{1+t^4}}{t}$$

b. $f(x) = \dfrac{1}{\cos(5x)} = \sec(5x)$

$$\frac{df}{dx} = \sec(5x)\tan(5x)(5)$$

c. $s(t) = \dfrac{\sin(2t)}{3+t^2}$

$$s'(t) = \frac{2\cos(2t)(3+t^2) - \sin(2t)(2t)}{(3+t^2)^2}$$

d. $f(x) = (x^2+1)e^x \sin(x)$

$$f'(x) = 2x(e^x \sin x) + (x^2+1)(e^x \sin x + e^x \cos x)$$

e. $g(z) = \cos(z^4+\pi)$

$$g'(z) = -\sin(z^4+\pi)(4z^3)$$

f. $f(t) = \dfrac{4}{t^{1/3}} + 2t^{1/3} + \sqrt{\dfrac{1}{3}}$

$$\frac{df}{dt} = 4\left(\frac{-1}{3}\right)t^{-4/3} + \frac{2}{3}t^{-2/3}$$

$$= -\frac{4}{3}t^{-4/3} + \frac{2}{3}t^{-2/3}$$

g. $f(x) = \dfrac{3x+7}{3\ln x + \ln 7}$

$$f'(x) = \frac{3(3\ln x + \ln 7) - (3x+7)\left(\frac{3}{x}\right)}{(3\ln x + \ln 7)^2}$$

h. $g(x) = \arcsin(e^x)$

$$\frac{dg}{dx} = \frac{1}{\sqrt{1-(e^x)^2}}\, e^x = \frac{e^x}{\sqrt{1-e^{2x}}}$$

i. $g(x) = (e^{2x} + e)\tan(x)$

$$g'(x) = (2e^{2x})\tan x + (e^{2x} + e)\sec^2 x$$

j. $h(x) = \ln(A + B\sin(x^2))$, where A and B are fixed constants

$$h'(x) = \frac{1}{A + B\sin(x^2)} \left(B\cos(x^2)\right)(2x)$$

$$= \frac{2Bx\cos(x^2)}{A + B\sin(x^2)}$$

k. $r(x) = \sec(x^2 + 1)$

$$\frac{dr}{dx} = \sec(x^2+1)\tan(x^2+1)\ (2x)$$

l. Compute $\dfrac{dy}{dx}$ if $xy + 2\sin y = e^{x+y}$. You must solve for $\dfrac{dy}{dx}$.

$$y + x\frac{dy}{dx} + 2\cos y\frac{dy}{dx} = e^{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\left(x + 2\cos y - e^{x+y}\right)\frac{dy}{dx} = e^{x+y} - y$$

$$\frac{dy}{dx} = \frac{e^{x+y} - y}{x + 2\cos y - e^{x+y}}$$