Name: Solutions ______ / 12

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- Circle your final answer.

Compute the derivatives of the following functions.

1. $f(x) = \ln(3) - \dfrac{1}{x^2}$

$$f'(x) = \frac{2}{x^3}$$

2. $y = e^{(ax^2)} + bx^3$, where a and b are fixed constants

$$y' = e^{ax^2} \cdot 2ax + 3bx^2$$

$$= 2axe^{ax^2} + 3bx^2$$

3. $g(x) = \left(\dfrac{1}{x} - x^2\right)(x-1)^3$

$$g'(x) = \left(\frac{1}{x} - x^2\right) \cdot 3(x-1)^2 + \left(-\frac{1}{x^2} - 2x\right)(x-1)^3$$

4. $h(y) = (y + \ln(y))^{3/2}$

$$\boxed{h'(y) = \frac{3}{2}(y + \ln y)^{1/2}\left(1 + \frac{1}{y}\right)}$$

5. $r(\theta) = \dfrac{1}{\cos(\theta)}$ $= \sec(\theta)$ $\boxed{r'(\theta) = \sec(\theta)\tan(\theta)}$

$$r'(\theta) = \frac{\cos\theta\,(0) - 1\,(-\sin\theta)}{\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta} \boxed{= \tan\theta\sec\theta}$$

6. $f(x) = \dfrac{\cos(\pi x)}{e^{3x} - 1}$

$$\boxed{f'(x) = \frac{(e^{3x} - 1)\cdot(-\sin(\pi x)\cdot\pi) - 3e^{3x}\cos(\pi x)}{(e^{3x} - 1)^2}}$$

7. $y = e^{-x}\tan(3x)\sin(x-\pi)$

$$y' = -e^{-x}\tan(3x)\sin(x-\pi) + e^{-x}\left[\sec^2(3x)\cdot 3\sin(x-\pi) + \tan(3x)\cos(x-\pi)\right]$$

8. $g(t) = \dfrac{t^2 - t^3 + 3t^{1/2}}{t^{1/2}}$ $= t^{3/2} - t^{5/2} + 3$

$$\boxed{g'(x) = \frac{3}{2}t^{1/2} - \frac{5}{2}t^{3/2}}$$

9. $f(x) = \ln(e^x + \sqrt{5})$

$$f'(x) = \frac{1}{e^x + \sqrt{5}} \cdot e^x$$

$$\boxed{= \frac{e^x}{e^x + \sqrt{5}}}$$

10. $f(x) = \left(\sqrt{1-x^2}\right)\arcsin(x)$

$$f'(x) = \frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)\arcsin x + \sqrt{1-x^2}\cdot\frac{1}{\sqrt{1-x^2}}$$

$$\boxed{= \frac{-x\arcsin x}{\sqrt{1-x^2}} + 1}$$

11. $s(t) = \tan\left(\ln\left(-t^3\right)\right)$

$$s'(t) = \sec^2(\ln(-t^3))\cdot\frac{1}{-t^3}\cdot(-3t^2)$$

$$\boxed{= \frac{3}{t}\sec^2(\ln(-t^3))}$$

12. Compute dy/dx if $\ln(y) + xy^2 = x^2 - 1$. You must solve for dy/dx.

$$\frac{1}{y}\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}\left(\frac{1}{y} + 2xy\right) = 2x - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{2x-y^2}{\frac{1}{y}+2xy}} \quad \text{or} \quad \boxed{\frac{dy}{dx} = \frac{2xy-y^3}{1+2xy^2}}$$